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# Interaction of collinear and noncollinear phonons in anharmonic scattering processes and their role in ultrasound absorption of fast quasi-transverse modes in cubic crystals

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## Abstract

The absorption of fast quasi-transverse modes during anharmonic scattering processes in cubic crystals with positive (Ge, Si, diamond and InSb) or negative (KCl and CaF<sub>2</sub>) anisotropies of the second-order elastic moduli is studied. Mechanisms underlying the relaxation of the fast quasi-transverse mode by two fast (the FFF mechanism) or two slow (the FSS) modes are discussed in the long-wavelength approximation. Angular dependences of the ultrasound absorption for the FFF, FSS and Landau–Rumer relaxation mechanisms are analyzed in terms of the anisotropic continuum model. The full absorption of the fast quasi-transverse mode is determined. The problem of the scattering of collinear and noncollinear phonons in cubic crystals and their role in the ultrasound absorption of the fast quasi-transverse modes is considered. It is shown that the FFF and FSS relaxation mechanisms are due to the cubic anisotropy of the crystals, leading to the interaction between noncollinear phonons. In crystals with a considerable anisotropy of the elastic energy (InSb and KCl), the total contribution of the FFF and FSS relaxation mechanisms to the full absorption is one to two orders of magnitude larger than the contribution from the Landau–Rumer mechanism, depending on the direction. Much of the dominance of the former relaxation mechanisms over the Landau–Rumer mechanism is explained by second-order elastic moduli. The role of the Landau–Rumer mechanism in ultrasound absorption may be considerable in cubic crystals with a smaller anisotropy of the elastic energy. It is demonstrated that when anharmonic scattering processes play the dominant role, the inclusion of one of the relaxation mechanisms (the Landau–Rumer mechanism or the FFF or FSS mechanisms of relaxation) is insufficient for the quantitative description of the anisotropy of the full absorption of the fast quasi-transverse modes in cubic crystals.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

The absorption of quasi-transverse ultrasound during anharmonic scattering processes in cubic crystals with a positive (Ge, Si, diamond and InSb) or a negative (KCl and NaCl) anisotropy of the second-order elastic moduli was studied in

our earlier works [1, 2]. The angular dependences of the absorptions for the Landau–Rumer relaxation mechanism [3], according to which the merging of a transverse and a longitudinal phonon produces a longitudinal phonon ( $T + L \rightarrow L$ ), were analyzed in [1]. The relaxation mechanisms of the slow quasi-transverse modes by two slow (the SSS mechanism) or

two fast (the SFF mechanism) modes were analyzed [2] in the long-wavelength approximation  $\hbar\omega_q^\lambda \ll k_B T$  ( $T$  being the temperature and  $\omega_q^\lambda$  the frequency of a phonon with a wavevector  $q$  and a polarization  $\lambda$ ). It was shown [2] that at sufficiently low temperatures these relaxation mechanisms can result in functional dependences of the absorption of the slow long-wavelength quasi-transverse modes in a form similar to that for the Landau–Rumer mechanism [3], i.e.  $\alpha_{\text{TTT}} \approx qT^4$ , and can compete with this mechanism. In accordance with the views established in the literature [1–7], the Landau–Rumer mechanism [3] is the main one for transverse phonons in normal three-phonon scattering processes. This relaxation mechanism was treated as the main mechanism for transverse thermal phonons in calculations of the lattice thermal conductivity [8–11]. However, our analysis [2] demonstrated that in crystals with a considerable anisotropy of the elastic energy (Ge, Si, InSb, KCl and NaCl) the total contribution from the SSS and SFF relaxation mechanisms to the ultrasound absorption is several times or one to two orders of magnitude larger than the contribution from the Landau–Rumer mechanism, depending on the direction. The dominance of the SSS and SFF relaxation mechanisms over the Landau–Rumer mechanism is explained, to a large extent, by the second-order elastic moduli. The role of the Landau–Rumer mechanism in ultrasound absorption is considerable in diamond crystals with a smaller anisotropy of the elastic energy. It has been shown that the SSS and SFF relaxation mechanisms are due to the cubic anisotropy of the crystals leading to the interaction of noncollinear phonons. Calculation of the ultrasound absorption for the SSS, SFF and Landau–Rumer relaxation mechanisms allowed us to determinate the full absorptions of slow quasi-transverse modes [2]. It was shown that other variants of the slow quasi-transverse mode relaxation, except those considered in [2], leading to the dependence of the Landau–Rumer type are unavailable.

The ultrasound absorption for anharmonic relaxation processes involving three transverse phonons was studied in [12–14]. However, it was shown [2] that the approximations used in [12–14] are inadequate for cubic crystals. Firstly, three-phonon scattering processes are calculated disregarding the influence of the cubic anisotropy on the phonon polarization: vibrational modes are assumed to be purely transverse modes as in isotropic media. In this case, the procedure used for averaging over the polarization vector directions in the matrix element is not correct for cubic crystals. Secondly, the ultrasound absorption is calculated for all symmetric directions, but the form of the conservation law adopted by the researchers allows the correct analysis of the [001] direction only. It is known [4, 15, 16] that quasi-longitudinal or quasi-transverse vibrations propagate in cubic crystals, while pure modes propagate only in symmetric directions such as [100], [110] and [111]. The analysis of the spectrum and the polarization of vibrational branches [16] demonstrated that the contribution of the transverse component to quasi-longitudinal vibrations in cubic crystals is small and can be neglected. Conversely, the contribution of the longitudinal components to the quasi-transverse modes is not small, and the longitudinal component of these modes should be taken into account in the

relaxation rates of quasi-transverse phonons when averaging over the polarization vectors. Thus, the approximations used in [12–14] can lead to large errors in calculating the relaxation rate of transverse phonons. In the present study the effect of cubic anisotropy on the spectrum and polarization of phonons is taken into account exactly in the anisotropic continuum model. The anisotropic continuum model is a convenient approximation for solving such problems [17, 18]. In this model the harmonic energy of cubic crystals is expressed as three second-order elastic moduli, whereas the anharmonic energy is treated in terms of six third-order elastic moduli. It should be noted that, for a considerable number of cubic crystals, the second- and third-order elastic moduli have been determined experimentally. Therefore, the phonon relaxation rates calculated in the framework of this model provide a reliable basis for the interpretation of experimental data on the ultrasonic absorption and phonon transport in cubic crystals. It should be noted that the anisotropy of the spectrum and the presence of degeneracy points in vibrational modes of transverse phonons lead to a new relaxation mechanism of the phonons in some anharmonic processes of scattering in cubic crystals as compared to isotropic media [19–21]. For example, the Herring mechanism [19], in which the fusion of a longitudinal phonon with a slow (ST) transverse phonon generates a fast (FT) transverse phonon, becomes possible. A new mechanism of transverse phonon relaxation in cubic crystals, according to which the fusion of a transverse phonon (slow or fast) with a slow transverse phonon generates a fast transverse phonon, has been investigated in [21]. This mechanism is similar to the Herring relaxation mechanism for longitudinal phonons [19] and can be referred to as the Herring mechanism for transverse phonons. These mechanisms are absent in isotropic media, because the transverse modes in them are degenerate. A new relaxation mechanism of transverse phonons [21], like the Herring mechanism for longitudinal phonons [19], leads to a dependence of the long-wavelength ultrasound absorption on the wavevector  $q$  and temperature  $T$  in the form  $\alpha_{\text{TTT}} \sim \alpha_{\text{LTT}} \sim q^2 T^3$ . As a rule, this dependence appears to be less effective in the long-wavelength limit than dependences of the Landau–Rumer type:  $\alpha_{\text{TTT}}^\lambda \sim qT^4$ . It contains an additional small parameter  $(\hbar\omega_q^\lambda/k_B T) \ll 1$  as compared to the Landau–Rumer mechanism.

In this study we restricted our analysis to the long-wavelength approximation and considered all anharmonic mechanisms responsible for relaxation of fast quasi-transverse modes and leading to a Landau–Rumer-type linear dependence of the ultrasound absorption on the wavevector. The analysis demonstrated that, apart from the Landau–Rumer mechanism, these relaxation processes involve relaxation of the fast quasi-transverse mode by two fast (the FFF mechanism) or two slow (the FSS mechanism) modes. Then it was possible to determine the total relaxation rate of fast quasi-transverse phonons in cubic crystals with positive (Ge, Si, diamond and InSb) and negative (KCl and CaF<sub>2</sub>) anisotropies of the second-order elastic moduli.

The estimates [4–7] showed that anharmonic relaxation processes involving three transverse phonons in isotropic media are ineffective. Firstly, transverse modes are degenerate

in the case of isotropic media and only collinear phonons can participate in these relaxation mechanisms. Secondly, as was shown in [22, 23], the matrix element of this interaction and, correspondingly, the relaxation rates and the ultrasound absorption of transverse phonons turn to zero. In what follows it will be shown that the use of the isotropic medium approximation [3–11, 18, 22, 23] for evaluating the probability of different scattering processes is inadequate for crystals of germanium, silicon, diamond and other semiconductor compounds, which have a cubic symmetry with a significant anisotropy of both harmonic and anharmonic energies. Firstly, the energy conservation law for the FSS and FFF relaxation mechanisms permits the interaction of both collinear and noncollinear phonons. Secondly, the analysis of the cubic anisotropy of the harmonic and anharmonic energies of the crystals demonstrated that the matrix element for the scattering of collinear phonons via the FFF and FSS relaxation mechanisms is nonzero only when the fast FT mode is a quasi-transverse mode whose longitudinal component is nonzero. The matrix element is zero if the fast FT mode is a purely transverse mode. Thus, the longitudinal component to the quasi-transverse modes should be considered when one deals with the ultrasound absorption of fast FT modes in cubic crystals.

In what follows we shall discuss in detail the problem of the scattering of collinear and noncollinear phonons in cubic crystals with positive (Ge, Si, diamond) or negative (KCl, CaF<sub>2</sub>) anisotropies of the second-order elastic moduli and their role in the ultrasound absorption of fast FT modes. The contributions of the FFF and FSS relaxation mechanisms to the ultrasound absorption will be compared with results obtained for the Landau–Rumer mechanism [1] and the full absorption of fast quasi-transverse modes will be determined. It will be demonstrated that other variants of the relaxation of fast quasi-transverse modes leading to dependences of the Landau–Rumer type are unavailable. Unlike in [12–14], we shall examine angular dependences of the ultrasound absorption for the two most important cases: (1) the wavevectors in the plane of the cube face and (2) the wavevectors in the diagonal plane. Then it will be possible to determine the directions in which the absorption of fast quasi-transverse modes is maximum and minimum.

## 2. Absorption of fast quasi-transverse modes in anharmonic scattering processes in cubic crystals

Let us consider the long-wave quasi-transverse ultrasound absorption  $\hbar\omega_q^\lambda \ll k_B T$  for which the Landau–Rumer regime is realized at sufficiently low temperatures if the inequality  $\omega_q^\lambda \tau^\lambda(\mathbf{q}, T) \gg 1$  is fulfilled ( $\tau^\lambda(\mathbf{q}, T) = 1/\nu^\lambda(\mathbf{q}, T)$ , where  $T$  is the temperature and  $\omega_q^\lambda$  is the frequency of a phonon with a wavevector  $q$  and a polarization  $\lambda$ ). If the above-mentioned inequalities are fulfilled, the ultrasonic wave absorption  $\alpha_\lambda(\mathbf{q}, T)$  with a wavevector  $q$  and a polarization  $\lambda$  is proportional to the full relaxation rate of phonons of the given polarization  $\nu^\lambda(\mathbf{q}, T)$  (see, e.g., [2, 4, 7]):

$$\alpha_\lambda(\mathbf{q}, T) = \frac{4.34\nu^\lambda(\mathbf{q}, T)}{S_\lambda(\mathbf{q})} (\text{dB cm}^{-1}), \quad (1)$$

where  $S_\lambda(\mathbf{q}) = S_\lambda(\theta, \varphi)$  is the phonon phase velocity, which depends on the angular variables  $\theta$  and  $\varphi$  of the vector  $q$ . In what follows we shall only consider the intervals of temperatures and wavevectors  $q$ , over which these inequalities hold. In the case under consideration the dominant contribution to the volume absorption of ultrasonic waves is due to the scattering by defects, including the isotopic scattering and normal processes of the phonon–phonon scattering (see, e.g., [4–7, 24]). Experimental studies of the ultrasound absorption [4, 24] demonstrated that the inequality  $\omega_q^\lambda \tau^\lambda(\mathbf{q}, T) \gg 1$  is fulfilled at sufficiently low temperatures. For example, it holds at temperatures below 50 K, 100 K and 300 K for germanium, silicon and diamond crystals, respectively. In these temperature ranges, Umklapp processes in phonon–phonon scattering for the long-wave ultrasound absorption are negligible. In the opposite limit case  $\omega_q^\lambda \tau^\lambda(\mathbf{q}, T) \ll 1$ , which takes place at higher temperatures, the Akhiezer mechanism of attenuation is dominant [25]. In this case, phonons can be considered to be quasiparticles moving in the smoothly varying field of a sonic wave and Umklapp processes in phonon–phonon scattering should be taken into account [4–7]. The scattering by defects is considered in [1]. The scattering processes TTL ( $T_1 + T_2 \rightarrow L_3$ ) considered in [22, 23] for the isotropic medium approximation may be effective for the thermal and high-frequency phonons. As shown by Maris [7], this relaxation mechanisms appears to be less effective in the long-wavelength limit than dependences of the Landau–Rumer type:  $\alpha_{\text{TTT}}^\lambda \sim qT^4$ . It contains additional small parameter  $(\hbar\omega_q^\lambda/k_B T)^3 \ll 1$  as compared to the Landau–Rumer mechanism. Here we shall restrict ourselves to the analysis of anharmonic processes of scattering which can result in functional dependences of the absorption of the upper vibrational modes in the long-wavelength approximation in a form similar to that for the Landau–Rumer mechanism. These variants of the relaxation of the fast transverse phonons in cubic crystals are the following processes of the merging of two transverse phonons, which is followed by the formation of a transverse phonon:

$$\begin{aligned} (1) \text{ FT}_1 + \text{FT}_2 &\rightarrow \text{FT}_3, & \omega_{q_1}^{t_1} &= \omega_{q_3}^{t_1} - \omega_{q_2}^{t_1}, \\ (2) \text{ FT}_1 + \text{ST}_2 &\rightarrow \text{ST}_3, & \omega_{q_1}^{t_1} &= \omega_{q_3}^{t_2} - \omega_{q_2}^{t_2}. \end{aligned} \quad (2)$$

The processes (1) involve three fast quasi-transverse phonons belonging to the upper vibrational branch (FT,  $\lambda = t_1$ ). In the processes (2) the fast transverse phonons interact with two slow quasi-transverse phonons belonging to the lower vibrational branch (ST,  $\lambda = t_2$ ). It has already been noted that collinear phonons can only participate in the processes (1) in isotropic media [4–7]. Processes like (1) can take place in isotropic media if the damping of phonon states is considered. The correct analysis of the effect of the damping of phonon states in the TTT mechanisms requires finding the total relaxation frequency of transverse thermal phonons, which is determined by all relaxation processes. The analysis of this mechanism [7] for isotropic media taking into account the damping of phonon states gives the wavevector-independent absorption, i.e.  $\alpha_{\text{TTT}}^\lambda \sim q^0 T^4 \nu(T)$ . In contrast, we shall show that in the case of the FFF mechanism (the processes (1)) the

energy conservation law is strictly fulfilled for noncollinear phonons too. They give dependences of the long-wavelength ultrasound absorption of the same form as those for the Landau–Rumer mechanism, i.e.  $\alpha_{\text{FFF}} \sim qT^4$ . Processes like (2) (the FSS mechanism) are analogous to processes of the relaxation of transverse phonons in the case of the Landau–Rumer mechanism. They give dependences of the long-wavelength ultrasound absorption of the form  $\alpha_{\text{FSS}} \sim qT^4$ , and can compete with the Landau–Rumer mechanism [2]. The energy of a scattered quasi-transverse phonon in the FSS mechanism equals the difference of the energies of thermal phonons belonging to one and the same transverse vibrational branch. It should be noted that this mechanism is impossible in isotropic media because the transverse branches are degenerate and the phonon spectrum is isotropic. Therefore, the higher the cubic anisotropy more efficient the FSS and FFF relaxation mechanisms are. In what follows we shall present calculations of the absorption of fast quasi-transverse modes for the processes (1) and (2) in cubic crystals. Then it will be possible to determine the full absorption of fast quasi-transverse modes in the long-wavelength approximation. Other variants of the relaxation of FT modes leading to a dependence of the Landau–Rumer type are absent. It will be shown that in many cubic crystals the FSS and FFF relaxation mechanisms makes the predominant contribution to the full absorption of the fast quasi-transverse mode.

The initial expression for the relaxation rate of phonons with a polarization  $\lambda$  has the form [9] (see also [4, 7, 18]):

$$v_{pN}(q_1, \lambda) = \frac{\pi \hbar^4}{(2\rho k_B T)^3} \frac{1}{V} \sum_{\substack{q_2, q_3 \\ \lambda_2, \lambda_3}} \frac{sh(\frac{z_1}{2}) \cdot \delta_{\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3, 0}}{z_1 z_2 z_3 sh(\frac{z_2}{2}) sh(\frac{z_3}{2})} \times \left| V_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3}^{\lambda \lambda_2 \lambda_3} \right|^2 \{ 2\delta(\omega_{\mathbf{q}_1}^\lambda + \omega_{\mathbf{q}_2}^{\lambda_2} - \omega_{\mathbf{q}_3}^{\lambda_3}) + \delta(\omega_{\mathbf{q}_1}^\lambda - \omega_{\mathbf{q}_2}^{\lambda_2} - \omega_{\mathbf{q}_3}^{\lambda_3}) \}. \quad (3)$$

Here  $\rho$  is the density,  $V$  is the normalization volume,  $T$  is the temperature, the polarization  $\lambda$  takes two values  $t_1$  and  $t_2$ , and  $z_n = \hbar \omega_{qn}^\lambda / k_B T$ . In expression (3) we shall only consider the first term in the braces, in which the merging of two transverse phonons produces a transverse phonon. Processes in the decomposition of a transverse phonon to two transverse phonons are not discussed since they can be considerable for thermal and high-frequency phonons. The expression for the matrix element  $V_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3}^{\lambda \lambda_2 \lambda_3}$ , determining the probabilities of various three-phonon scattering processes in terms of the elastic moduli of the second and third order, was obtained in [26]:

$$V_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3}^{\lambda_1 \lambda_2 \lambda_3} = \tilde{c}_{111} \sum_i e_{1i} q_{1i} e_{2i} q_{2i} e_{3i} q_{3i} + c_{123} (\mathbf{e}_1 \mathbf{q}_1) (\mathbf{e}_2 \mathbf{q}_2) (\mathbf{e}_3 \mathbf{q}_3) + \tilde{c}_{112} \sum_i [( \mathbf{e}_1 \mathbf{q}_1 ) e_{2i} q_{2i} e_{3i} q_{3i} + ( \mathbf{e}_2 \mathbf{q}_2 ) e_{1i} q_{1i} e_{3i} q_{3i} + ( \mathbf{e}_3 \mathbf{q}_3 ) e_{1i} q_{1i} e_{2i} q_{2i}] + c_{144} [ ( \mathbf{e}_1 \mathbf{q}_1 ) ( \mathbf{e}_2 \mathbf{q}_3 ) ( \mathbf{e}_3 \mathbf{q}_2 ) + ( \mathbf{e}_2 \mathbf{q}_2 ) ( \mathbf{e}_1 \mathbf{q}_3 ) ( \mathbf{e}_3 \mathbf{q}_1 ) + ( \mathbf{e}_3 \mathbf{q}_3 ) ( \mathbf{e}_1 \mathbf{q}_2 ) ( \mathbf{e}_2 \mathbf{q}_1 ) ] + (c_{12} + c_{144}) [ ( \mathbf{e}_1 \mathbf{q}_1 ) ( \mathbf{e}_2 \mathbf{e}_3 ) ( \mathbf{q}_3 \mathbf{q}_2 ) + ( \mathbf{e}_2 \mathbf{q}_2 ) ( \mathbf{e}_1 \mathbf{e}_3 ) ( \mathbf{q}_3 \mathbf{q}_1 ) + ( \mathbf{e}_3 \mathbf{q}_3 ) ( \mathbf{e}_1 \mathbf{e}_2 ) ( \mathbf{q}_1 \mathbf{q}_2 ) ] + c_{456} [ ( \mathbf{e}_1 \mathbf{q}_3 ) ( \mathbf{e}_2 \mathbf{q}_1 ) ( \mathbf{e}_3 \mathbf{q}_2 ) + ( \mathbf{e}_1 \mathbf{q}_2 ) ( \mathbf{e}_2 \mathbf{q}_3 ) ( \mathbf{e}_3 \mathbf{q}_1 ) ] + (c_{44} + c_{456}) [ ( \mathbf{e}_1 \mathbf{q}_2 ) ( \mathbf{q}_1 \mathbf{q}_3 ) ( \mathbf{e}_2 \mathbf{e}_3 )$$

$$+ ( \mathbf{e}_2 \mathbf{q}_3 ) ( \mathbf{q}_1 \mathbf{q}_2 ) ( \mathbf{e}_1 \mathbf{e}_3 ) + ( \mathbf{e}_3 \mathbf{q}_1 ) ( \mathbf{q}_2 \mathbf{q}_3 ) ( \mathbf{e}_1 \mathbf{e}_2 ) + ( \mathbf{e}_1 \mathbf{q}_3 ) ( \mathbf{q}_1 \mathbf{q}_2 ) ( \mathbf{e}_2 \mathbf{e}_3 ) + ( \mathbf{e}_2 \mathbf{q}_1 ) ( \mathbf{q}_2 \mathbf{q}_3 ) ( \mathbf{e}_1 \mathbf{e}_3 ) + ( \mathbf{e}_3 \mathbf{q}_2 ) ( \mathbf{q}_1 \mathbf{q}_3 ) ( \mathbf{e}_1 \mathbf{e}_2 ) ] + \tilde{c}_{155} \sum_i \{ e_{1i} e_{2i} e_{3i} ( q_{1i} ( \mathbf{q}_2 \mathbf{q}_3 ) + q_{2i} ( \mathbf{q}_1 \mathbf{q}_3 ) + q_{3i} ( \mathbf{q}_1 \mathbf{q}_2 ) ) + e_{1i} q_{1i} [ e_{2i} q_{3i} ( \mathbf{e}_3 \mathbf{q}_2 ) + e_{3i} q_{2i} ( \mathbf{e}_2 \mathbf{q}_3 ) ] + e_{2i} q_{2i} [ e_{1i} q_{3i} ( \mathbf{e}_3 \mathbf{q}_1 ) + e_{3i} q_{1i} ( \mathbf{e}_1 \mathbf{q}_3 ) ] + e_{3i} q_{3i} [ e_{1i} q_{2i} ( \mathbf{e}_2 \mathbf{q}_1 ) + e_{2i} q_{1i} ( \mathbf{e}_1 \mathbf{q}_2 ) ] \} + [ \tilde{c}_{155} - \Delta C ] \times \sum_i q_{1i} q_{2i} q_{3i} [ e_{1i} ( \mathbf{e}_2 \mathbf{e}_3 ) + e_{2i} ( \mathbf{e}_1 \mathbf{e}_3 ) + e_{3i} ( \mathbf{e}_1 \mathbf{e}_2 ) ], \quad (4)$$

where

$$\begin{aligned} \tilde{c}_{112} &= c_{112} - c_{123} - 2c_{144}, \\ \tilde{c}_{155} &= c_{155} - c_{144} - 2c_{456}, \\ \tilde{c}_{111} &= c_{111} - 3c_{112} + 2c_{123} + 12c_{144} - 12c_{155} + 16c_{456}, \\ \Delta C &= c_{12} + 2c_{44} - c_{11}. \end{aligned} \quad (5)$$

Here  $c_{ij}$  and  $c_{ijk}$  are the thermodynamic moduli of second and third order in the normalization introduced by Brugger [27] and  $\mathbf{e}_{1,2,3}$  stands for polarization vectors. The contributions containing the elastic moduli  $\tilde{c}_{111}$ ,  $\tilde{c}_{112}$ ,  $\tilde{c}_{155}$ ,  $\Delta C$  are typical of cubic crystals and distinguish them from the isotropic medium. Equality to zero of these moduli provides a transition to the isotropic medium model. In this case one can obtain the expression for the anharmonic energy of the isotropic medium obtained by Tucker and Rampton (see formula (4.22) of [5]).

The relaxation rate of phonons for the TTT relaxation mechanisms is calculated in the anisotropic continuum model. First of all, we determine the spectrum of phonons with a polarization  $\lambda$  and a wavevector much smaller than the Debye wavevector  $q_d$  in this model:

$$\omega_q^\lambda = S_\lambda(\theta, \varphi) q. \quad (6)$$

The spectrum anisotropy is determined by the anisotropy of the phase velocity  $S_\lambda(\theta, \varphi)$ , which depends on the angles  $\theta$  and  $\varphi$  of the vector  $\mathbf{q}$ . In the system of coordinates connected with the cube edges we have [16]:

$$\begin{aligned} S_\lambda(\theta, \varphi) &= \sqrt{\frac{c_{44}}{\rho}} \left( 1 + \frac{c_{11} - c_{44}}{c_{44}} \left( \frac{1}{3} + Z_\lambda \right) \right)^{1/2}, \\ Z_{t_1, t_2} &= \frac{2}{3} r \cos \left( \frac{Q}{3} \mp \frac{2\pi}{3} \right), \quad Q = \arccos p, \\ p &= \left\{ \frac{1 + 4.5(k^2 - 1)\xi + 13.5\eta(k - 1)^2(2k + 1)}{r^3} \right\}, \\ r &= \sqrt{1 + 3(k^2 - 1)\xi}, \quad k = \frac{c_{12} + c_{44}}{c_{11} - c_{44}} \end{aligned} \quad (7)$$

where  $\xi = n_1^2 n_2^2 + n_1^2 n_3^2 + n_2^2 n_3^2$  and  $\eta = n_1^2 n_2^2 n_3^2$  are cubic harmonics and  $\mathbf{n} = \mathbf{q}/q = \{ \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta \}$  is a unit wavevector of a phonon. The indices  $t_1$  and  $t_2$  correspond to the ‘fast’ (the upper) and the ‘slow’ (the lower) transverse vibrational modes.

In the exact expression for the matrix element of (4) we will take into account the terms that are linear in longitudinal components of quasi-transverse vibrations,



while the terms proportional to quadratic combinations of  $(\mathbf{e}_1\mathbf{n}_1)$ ,  $(\mathbf{e}_2\mathbf{n}_2)$ ,  $(\mathbf{e}_3\mathbf{n}_3)$  will be neglected. The error of this approximation is about 1% in Ge, InSb, GaSb and GaAs, and less than 1% in Si and diamond. Then for the square of the matrix element in the long-wavelength approximation ( $q_2 \cong q_3$ ,  $\mathbf{n}_2 \cong \mathbf{n}_3$  and  $\mathbf{e}_2 \cong \mathbf{e}_3$ ) we have for the relaxation processes (1) and (2):

$$I_{\text{TTT}}^{\lambda\lambda_2\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) = \left(2\frac{q_1q_2^3}{q_3}\right)^{-2} \left|V_{\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3}^{\lambda\lambda_2\lambda_3}\right|^2 = \frac{1}{4} \left\{ \tilde{c}_{111} \times \sum_i e_{1i}e_{2i}^2n_{1i}n_{2i}^2 + \tilde{c}_{112} \sum_i [2e_{1i}e_{2i}n_{1i}n_{2i}(e_2n_2) + e_{2i}^2n_{2i}^2(\mathbf{e}_1\mathbf{n}_1)] + \tilde{c}_{155} \sum_i [e_{1i}e_{2i}^2(n_{1i} + 2n_{2i} \cos \theta_{12}) + 2e_{1i}e_{2i}n_{2i}[n_{1i}(\mathbf{e}_2\mathbf{n}_2) + n_{2i}(\mathbf{e}_2\mathbf{n}_1)] + 2e_{2i}^2n_{1i}n_{2i}(\mathbf{e}_1\mathbf{n}_2)] + (\tilde{c}_{155} - \Delta C) \sum_i [n_{1i}n_{2i}^2(e_{1i} + 2e_{2i}(\mathbf{e}_1\mathbf{e}_2))] + 2(c_{144} + c_{456})(\mathbf{e}_2\mathbf{n}_2)(\mathbf{e}_2\mathbf{n}_1)(\mathbf{e}_1\mathbf{n}_2) + (c_{12} + c_{144}) \times [(\mathbf{e}_1\mathbf{n}_1) + 2(\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_2\mathbf{n}_2) \cos \theta_{12}] + 2(c_{44} + c_{456}) \times [(\mathbf{e}_1\mathbf{n}_2) \cos \theta_{12} + (\mathbf{e}_1\mathbf{e}_2)[(\mathbf{e}_2\mathbf{n}_1) + (\mathbf{e}_2\mathbf{n}_2) \cos \theta_{12}]] \right\}^2, \quad (8)$$

$$\cos \theta_{12} = (\mathbf{n}_1\mathbf{n}_2) = \sin \theta_1 \cos(\varphi_2 - \varphi_1) \sin \theta_2 + \cos \theta_1 \cos \theta_2.$$

Here  $\theta_1, \varphi_1$  and  $\theta_2, \varphi_2$  are the angular variables of the vectors  $\mathbf{q}_1$  and  $\mathbf{q}_2$ . The polarization vectors of phonons in different vibrational branches in the formula (8) are defined by the expressions [16]

$$e_j^\lambda = \frac{1}{A_\lambda} \left\{ \frac{n_j}{\psi_j^\lambda} \right\} \quad A_\lambda = \pm \sqrt{\sum_j \frac{n_j^2}{(\psi_j^\lambda)^2}} \quad (9)$$

$$(\mathbf{e}^\lambda\mathbf{n}) = \frac{1}{A_\lambda} \sum_j \frac{n_j^2}{\psi_j^\lambda}, \quad \psi_j^\lambda = \frac{1}{3} + Z_\lambda + (k-1)n_j^2.$$

It is seen from (7) and (9) that just the parameter  $k-1$  characterizes the influence of the elastic anisotropy on the spectrum and polarization vectors of vibrational modes in cubic crystals. When changing to the isotropic medium model, which is discussed in [22, 23], the terms containing the third-order elastic moduli  $\tilde{c}_{111}$ ,  $\tilde{c}_{155}$ ,  $\Delta C$  and  $(\mathbf{e}_2\mathbf{n}_2)$ ,  $(\mathbf{e}_1\mathbf{n}_1)$ , vanish and, hence, only the last term remains in (8). It determines the matrix element for the isotropic medium:

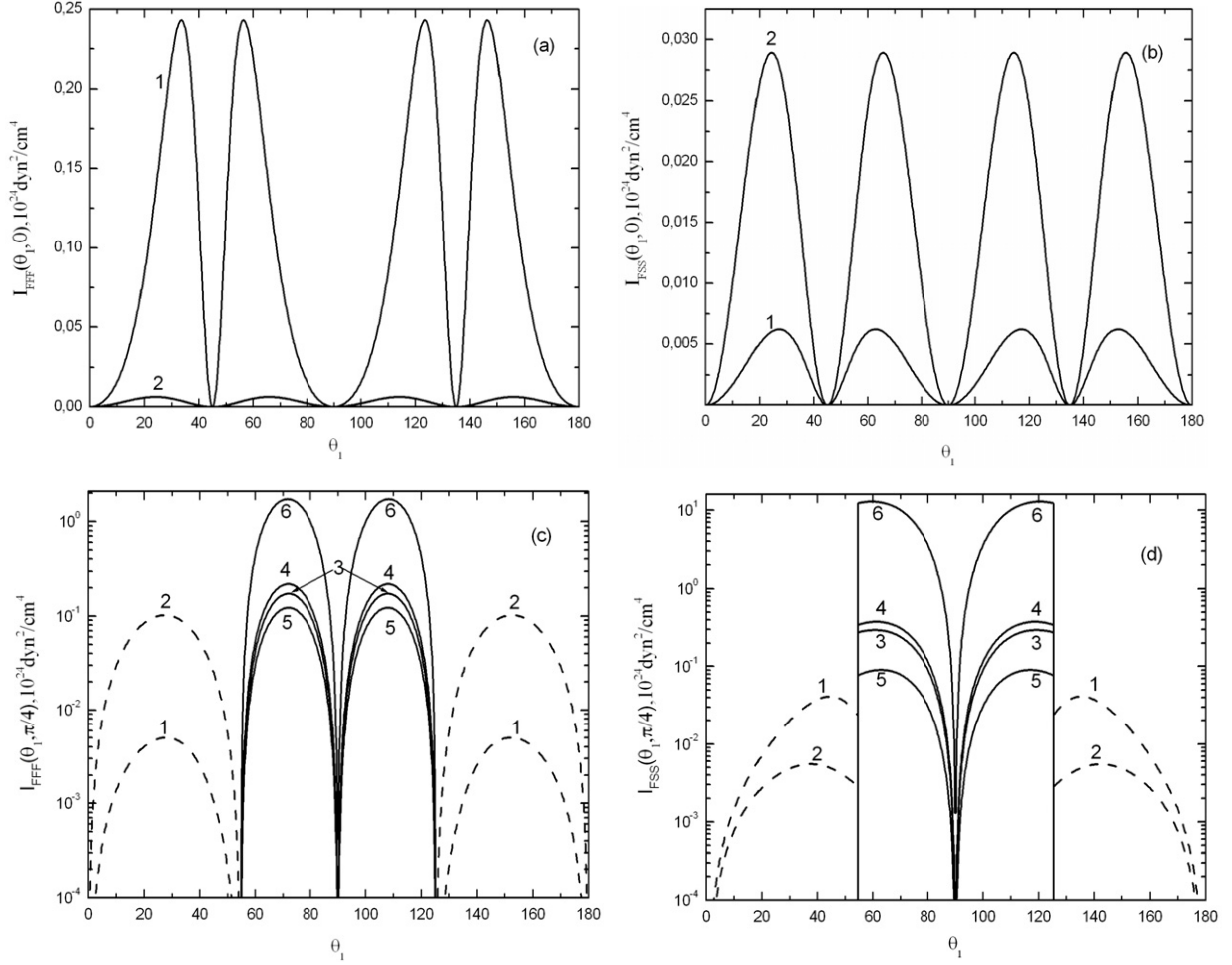
$$I_{\text{TTT}}^{\lambda\lambda_2\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) = \frac{1}{4} [2(c_{44} + c_{456})[(\mathbf{e}_1\mathbf{n}_2) \cos \theta_{12} + (\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_2\mathbf{n}_1)]]^2. \quad (10)$$

The result obtained in [22, 23] becomes obvious from this expression. For the scattering of collinear phonons  $\mathbf{n}_1 = \mathbf{n}_2$  and the matrix element identically turns to zero because isotropic media pass pure modes for which  $(\mathbf{e}_2\mathbf{n}_2) = (\mathbf{e}_1\mathbf{n}_1) = 0$ . Thus, the square of the matrix element for the scattering of collinear phonons in isotropic media identically turns to zero in an arbitrary direction of the wavevector of a phonon. Therefore, the TTT relaxation mechanisms in isotropic media can contribute to ultrasound absorption only because of the small-angle scattering of phonons if the damping of phonon states is considered.

The situation is considerably different for cubic crystals. The consideration of the cubic anisotropy of the harmonic and anharmonic energies of the crystals leads to nonzero values of the matrix element (8) for the scattering of collinear phonons via the FFF and FSS relaxation mechanisms only when the fast FT mode is a quasi-transverse mode whose longitudinal component is nonzero. The square of the matrix element is zero if the fast FT mode is a purely transverse mode. For example, in all the crystals of the first group the fast FT mode for the wavevectors in the cube face plane is a purely transverse mode with the polarization vector being perpendicular to the cube face under consideration (see [16]). It is easy to check that for this mode  $e_{1i}n_{1i} = 0$  at all  $i$  and, therefore, expression (8) for the FFF and FSS relaxation mechanisms becomes zero at all values of the angle  $\theta_1$  for the scattering of collinear phonons ( $\mathbf{n}_1 = \mathbf{n}_2$ ). Conversely, in all the crystals of the second group (including KCl and CaF<sub>2</sub>) the fast FT mode for the wavevectors in the cube face plane is a quasi-transverse mode with the polarization vector being in the cube face plane (see [16]). Therefore, the squares of the matrix element for the scattering of collinear phonons in the FFF ( $I_{\text{TTT}}^{t_1t_1t_1}(\theta_1, 0, \theta_1, 0) \equiv I_{\text{FFF}}(\theta_1, 0)$ ) and FSS ( $I_{\text{TTT}}^{t_1t_2t_2}(\theta_1, 0, \theta_1, 0) = I_{\text{FSS}}(\theta_1, 0)$ ) relaxation mechanisms are nonzero at all values of the angle  $\theta_1$ . As can be seen from figures 1(a) and (b), they turn to zero in the [001] and [101] directions in all the crystals of the second group. This is because the fast FT mode is a purely transverse mode in these directions.

In the case of the wavevectors lying in the diagonal plane ( $\varphi_1 = \pi/4$ ), the square of the matrix element  $I_{\text{TTT}}^{t_1t_1t_1}(\theta_1, \pi/4, \theta_1, \pi/4) \equiv I_{\text{FFF}}(\theta_1, \pi/4)$  is nonzero in the crystals of the first group (Ge, Si, diamond and InSb) at the angles  $\theta_{111} < \theta_1 < \pi - \theta_{111}$  ( $\theta_{111}$  being the angle between the  $z$ -axis and the [111] direction) (see figure 1(c)). In this case, the fast FT mode is a quasi-transverse mode with the polarization vector in the diagonal plane. At the angles  $0 < \theta_1 < \theta_{111}$  and  $\pi - \theta_{111} < \theta_1 < \pi$  the fast FT mode is a pure mode with the polarization vector being perpendicular to the diagonal plane and the function  $I_{\text{FFF}}(\theta_1, \pi/4)$  turns to zero (see figure 1(c)). In the crystals of the second group the square of the matrix element  $I_{\text{FFF}}(\theta_1, \pi/4)$  is nonzero at the angles  $0 < \theta_1 < \theta_{111}$  and  $\pi - \theta_{111} < \theta_1 < \pi$  (see figure 1(c)) (the mode  $t_1$  is a quasi-transverse mode with the polarization vector lying in the diagonal plane) and is zero at the angles  $\theta_{111} < \theta_1 < \pi - \theta_{111}$  (the mode  $t_1$  is a pure mode with the polarization vector being perpendicular to the diagonal plane) (see [16]). The same situation takes place for the FSS relaxation mechanism. With the wavevectors in the diagonal plane ( $\varphi_1 = \pi/4$ ), the square of the matrix element  $I_{\text{TTT}}^{t_1t_2t_2}(\theta_1, \pi/4, \theta_1, \pi/4) = I_{\text{FSS}}(\theta_1, \pi/4)$  for the FSS relaxation mechanism is nonzero at the angles  $\theta_{111} < \theta_1 < \pi - \theta_{111}$  in the crystals of the first group and at the angles  $0 < \theta_1 < \theta_{111}$  and  $\pi - \theta_{111} < \theta_1 < \pi$  in the crystals of the second group (see figure 1(d)). The above analysis suggests that the angular dependences of the square of the matrix element are qualitatively different for the scattering of collinear phonons in cubic crystals with positive and negative anisotropies of the elastic energy.

Let us consider the energy conservation law in the FSS and FFF relaxation mechanisms. The equality of the function



**Figure 1.** Angular dependences of the square of the matrix element during the scattering of collinear phonons in the FFF ((a), (c)) and FSS ((b), (d)) relaxation mechanisms for sound wavevectors in the cube face plane ((a), (b)) in KCl (curve 1) and CaF<sub>2</sub> (curve 2) and for sound wavevectors in the diagonal plane ((c), (d)) in Ge (curve 3), Si (curve 4), InSb (curve 5), diamond (curve 6), KCl (curve 1) and CaF<sub>2</sub> (curve 2).

$\Omega_{\lambda\lambda_2}$  to zero determines the energy conservation law for the TTT mechanisms:

$$\Omega_{\lambda\lambda_2} = (\omega_{q_1}^\lambda + \omega_{q_2}^{\lambda_2} - \omega_{|q_1+q_2|}^{\lambda_2}) = \omega_{q_1}^\lambda \times \left[ 1 - \frac{\omega_{|q_1+q_2|}^{\lambda_2} - \omega_{q_2}^{\lambda_2}}{\omega_{q_1}^{\lambda_2}} \right]. \quad (11)$$

For an arbitrary direction of the phonon wavevectors  $\mathbf{q}_2$  and  $\mathbf{q}_3$  at  $q_1 \ll q_2, q_3$  this function can be expressed in terms of the group velocity of phonons  $V_{g2}^{\lambda_2}$ :

$$\Omega_{\lambda\lambda_2} = \omega_{q_1}^\lambda W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2),$$

$$W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) = 1 - \frac{1}{S_\lambda(\theta_1, \varphi_1)} (V_{g2}^{\lambda_2}(\theta_2, \varphi_2) \cdot \mathbf{n}_1). \quad (12)$$

The group velocity of phonons can be written in the form

$$V_{g2}^{\lambda_2}(\theta_2, \varphi_2) = S_{\lambda_2}(\theta_2, \varphi_2) \{ \mathbf{n}_2 + S_{\theta_2}^{\lambda_2} \mathbf{e}_{\theta_2} + S_{\varphi_2}^{\lambda_2} \mathbf{e}_{\varphi_2} \}, \quad (13)$$

where

$$\mathbf{n}_2 = \{ \sin \theta_2 \cos \varphi_2, \sin \theta_2 \sin \varphi_2, \cos \theta_2 \},$$

$$\mathbf{e}_{\theta_2} = \{ \cos \theta_2 \cos \varphi_2, \cos \theta_2 \sin \varphi_2, -\sin \theta_2 \},$$

$$\mathbf{e}_{\varphi_2} = \{ -\sin \varphi_2, \cos \varphi_2, 0 \},$$

$$S_{\theta_2}^{\lambda_2} = \frac{1}{S_{\lambda_2}} \frac{\partial S_{\lambda_2}}{\partial \theta_2} = \frac{\partial}{\partial \theta_2} \ln S_{\lambda_2}, \quad S_{\varphi_2}^{\lambda_2} = \frac{1}{\sin \theta_2} \frac{1}{S_{\lambda_2}} \frac{\partial S_{\lambda_2}}{\partial \varphi_2}.$$

The functions  $S_\theta^\lambda(\theta, \varphi)$  and  $S_\varphi^\lambda(\theta, \varphi)$  are determined by the following expressions:

$$\begin{aligned} S_\theta^\lambda(\theta, \varphi) &= \frac{1}{S_\lambda} \frac{\partial S_\lambda}{\partial \theta} = \frac{1}{S_\lambda} \left( \frac{\partial S_\lambda}{\partial \xi} \frac{\partial \xi}{\partial \theta} + \frac{\partial S_\lambda}{\partial \eta} \frac{\partial \eta}{\partial \theta} \right), \\ S_\varphi^\lambda(\theta, \varphi) &= \frac{1}{\sin \theta \cdot S_\lambda} \left( \frac{\partial S_\lambda}{\partial \xi} \frac{\partial \xi}{\partial \varphi} + \frac{\partial S_\lambda}{\partial \eta} \frac{\partial \eta}{\partial \varphi} \right) \quad (14) \\ \frac{1}{S_\lambda} \frac{\partial S_\lambda}{\partial \xi} &= \left[ \left( \frac{S_{100}^t}{S_\lambda} \right)^2 \frac{c_{11} - c_{44}}{2c_{44}} Z_\lambda \right] \frac{1.5(k^2 - 1)}{r^2} \\ &\times \left\{ 1 - \frac{1.5(k^2 - 1)\xi + 13.5(k - 1)^2(1 + 2k)}{(1 - q^2)^{1/2}(r)^3} \right\} \\ &\times \operatorname{tg} \left( \frac{Q}{3} \mp \frac{2\pi}{3} \right) \\ \frac{1}{S_\lambda} \frac{\partial S_\lambda}{\partial \eta} &= \left[ \left( \frac{S_{100}^t}{S_\lambda} \right)^2 \frac{c_{11} - c_{44}}{2c_{44}} Z_\lambda \right] \frac{4.5(k - 1)^2(1 + 2k)}{(1 - q^2)^{1/2}(r)^3} \\ &\times \operatorname{tg} \left( \frac{Q}{3} \mp \frac{2\pi}{3} \right), \quad S_{100}^t = \left( \frac{c_{44}}{\rho} \right)^{1/2}, \end{aligned}$$

$$\frac{\partial \xi}{\partial \theta} = \sin 2\theta [\cos 2\theta + 0.5(\sin \theta)^2 (\sin 2\varphi)^2],$$

$$\frac{\partial \xi}{\partial \varphi} = 0.5(\sin \theta)^4 \sin 4\varphi,$$

$$\frac{\partial \eta}{\partial \theta} = \frac{1}{4} \sin 2\theta (\sin \theta)^2 [3(\cos \theta)^2 - 1] \sin^2 2\varphi,$$

$$\frac{\partial \eta}{\partial \varphi} = \frac{1}{2} (\sin \theta)^4 (\cos \theta)^2 \sin 4\varphi.$$

The expressions for  $Z_\lambda$ ,  $q$  and  $r$  are given by formulae (7). It is easy to verify that the vectors  $\mathbf{n}_2$ ,  $\mathbf{e}_{\theta_2}$  and  $\mathbf{e}_{\varphi_2}$  form a mutually orthogonal system of unit vectors. In an arbitrary direction of the unit wavevectors of phonons  $\mathbf{n}_1$  and  $\mathbf{n}_2$  we have

$$\begin{aligned} W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) &= 1 - \frac{S_{\lambda_2}(\theta_2, \varphi_2)}{S_\lambda(\theta_1, \varphi_1)} \{\cos \theta_{12} + S_{\theta_2}^{\lambda_2}(\theta_2, \varphi_2) \\ &\times [\sin \theta_1 \cos \theta_2 \cos(\varphi_2 - \varphi_1) - \sin \theta_2 \cos \theta_1] \\ &+ S_{\varphi_2}^{\lambda_2}(\theta_2, \varphi_2) [\sin \theta_1 \sin(\varphi_1 - \varphi_2)]\}. \end{aligned} \quad (15)$$

In the [001] direction ( $\theta_1 = 0$ ,  $\varphi_1 = 0$ ) it follows from (15) that

$$W_{\lambda\lambda_2}(0, 0, \theta_2, \varphi_2) = 1 - S_{\lambda_2}(\theta_2, \varphi_2) [\cos \theta_2 - S_{\theta_2}^{\lambda_2} \sin \theta_2] / S_{100}^t. \quad (16)$$

Expression (16) is used in [12–14] for calculating the energy conservation contours in all symmetric directions ([001], [101] and [111]). However, it holds for the [001] direction only. For example, in the [101] direction ( $\theta_1 = \pi/4$ ,  $\varphi_1 = 0$ ) it follows from (15) that

$$\begin{aligned} W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) &= 1 - \frac{S_{\lambda_2}(\theta_2, \varphi_2)}{\sqrt{2}S_\lambda(\theta_1, \varphi_1)} \{\sin \theta_2 \cos \varphi_2 \\ &+ \cos \theta_2 + S_{\theta_2}^{\lambda_2}(\theta_2, \varphi_2) (\cos \theta_2 \cos \varphi_2 - \sin \theta_2) \\ &- S_{\varphi_2}^{\lambda_2} \sin \varphi_2\}. \end{aligned} \quad (17)$$

Obviously, expressions (16) and (17) for the function  $W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  are considerably different. Its use for the other symmetric directions, as was done in [12–14], is erroneous. Therefore the analysis of the energy conservation law and the ultrasound absorptions [12–14] in the [101] and [111] directions is incorrect.

In the long-wavelength limit  $\hbar\omega_q^\lambda \ll k_B T$  ( $q_1 \ll q_2$ ,  $q_3$ ) at temperatures much lower than the Debye temperature the integral over  $z_2$  in (3) is calculated exactly and for the ultrasound absorption in the FFF ( $\lambda = t_1$  and  $\lambda_2 = \lambda_3 = t_1$ ) and FSS ( $\lambda = t_1$  and  $\lambda_2 = \lambda_3 = t_2$ ) processes we have

$$\alpha_{\text{FTT}}(\theta_1, \varphi_1, T) = A_{\text{FTT}} z_1 T^5, \quad z_1 = \frac{\hbar\omega_{q_1}^t}{k_B T},$$

$$A_{\text{FTT}} = A_{\text{FFF}} + A_{\text{FSS}} = A_{0F} J_{\text{FTT}}(\theta_1, \varphi_1) \text{ (dB cm}^{-1}\text{K}^{-5}\text{)}, \quad (18)$$

$$J_{\text{FTT}}(\theta_1, \varphi_1) = J_{\text{FFF}}(\theta_1, \varphi_1) + J_{\text{FSS}}(\theta_1, \varphi_1),$$

$$A_{0F} = \frac{4.34\pi^3 k_B^5}{15\hbar^4 \rho^3 (S_{t_1}(\theta_1, \varphi_1))^2 (S_{100}^t)^8}, \quad S_{100}^t = \left(\frac{c_{44}}{\rho}\right)^{1/2} \quad (19)$$

$$\begin{aligned} J_{\text{FTT}}(\theta_1, \varphi_1) &= \sum_{\lambda_2} \int_{-1}^1 dx \frac{1}{\pi} \int_0^{2\pi} d\varphi_2 \delta(F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)) \\ &\times \frac{I_{\text{TTT}}^{\lambda\lambda_2\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)}{(\tilde{S}_{\lambda_2}(\theta_2, \varphi_2))^8}, \quad x = \cos \theta_2 \end{aligned} \quad (20)$$

$$\begin{aligned} F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) &= \cos \theta_{12} - \left( \frac{S_{t_1}(\theta_1, \varphi_1)}{S_{\lambda_2}(\theta_2, \varphi_2)} \right. \\ &\left. - \Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) \right), \end{aligned}$$

$$\tilde{S}_{\lambda_2}(\theta_2, \varphi_2) = \frac{S_{\lambda_2}(\theta_2, \varphi_2)}{S_{100}^t}. \quad (21)$$

The equality  $F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) = 0$  determines the energy conservation contours for the considered relaxation mechanisms. The quantities  $F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  and  $\Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  introduced by us can be presented in terms of the above-defined quantities  $W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  and the group velocity of phonons in the form:

$$\begin{aligned} F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) &= -\frac{S_{\lambda_1}}{S_{\lambda_2}} W_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) \\ \Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2) &= S_{\theta_2}^{\lambda_2}(\mathbf{e}_{\theta_2} \mathbf{n}_1) + S_{\varphi_2}^{\lambda_2}(\mathbf{e}_{\varphi_2} \mathbf{n}_1) \\ &= S_{\theta_2}^{\lambda_2}(\theta_2, \varphi_2) [\sin \theta_1 \cos \theta_2 \cos(\varphi_2 - \varphi_1) - \sin \theta_2 \cos \theta_1] \\ &+ S_{\varphi_2}^{\lambda_2}(\theta_2, \varphi_2) [\sin \theta_1 \sin(\varphi_2 - \varphi_1)]. \end{aligned} \quad (22)$$

The use of the function  $F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  and  $\Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  is convenient for study of the energy conservation law in the FFF and FSS relaxation mechanisms for the interaction of collinear and noncollinear phonons.

Consider the restrictions related to the energy conservation law with respect to the scattering of collinear phonons in cubic crystals. Notice that in isotropic media the spectrum of transverse phonons is isotropic and the energy conservation law allows the scattering of collinear phonons at an arbitrary direction of the wavevector. For the scattering of collinear phonons  $\mathbf{n}_1 = \mathbf{n}_2$ , and it follows from (22) that the function  $\Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_1, \varphi_1)$  identically turn to zero at an arbitrary direction of the vector  $\mathbf{n}_1$ . Therefore the fulfilment of the energy conservation law for these processes reduces to the condition

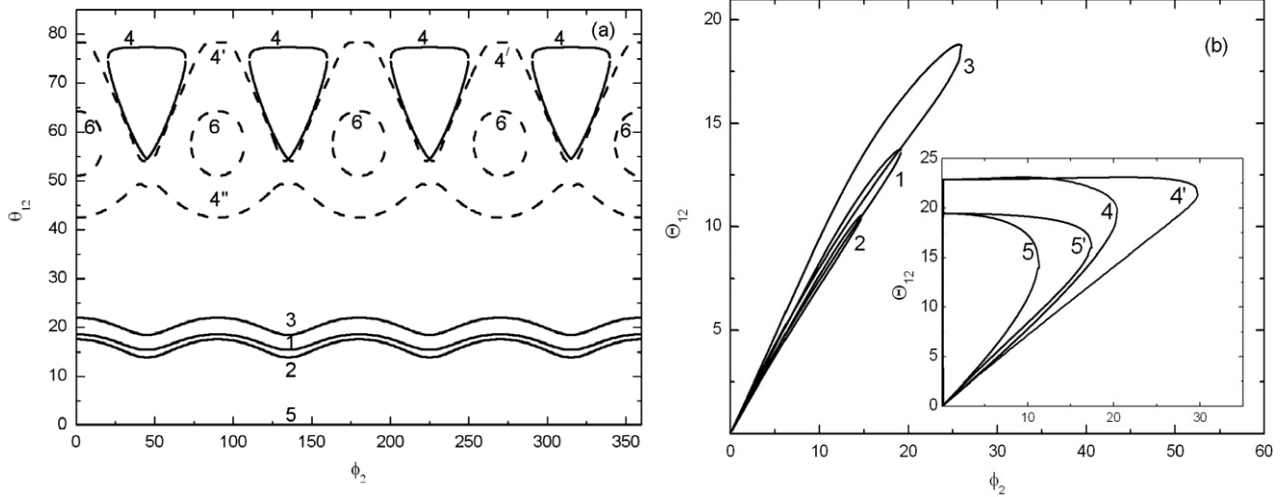
$$F_{\lambda\lambda_2}(\theta_1, \varphi_1, \theta_1, \varphi_1) = 1 - \frac{S_\lambda(\theta_1, \varphi_1)}{S_{\lambda_2}(\theta_1, \varphi_1)} = 0. \quad (23)$$

It is seen from (23) that for the FFF and SSS mechanisms the polarizations of interacting phonons are equal ( $\lambda = \lambda_2$ ) and the energy conservation law allows the scattering of collinear phonons at all directions of the vector  $\mathbf{n}_1$ . For the FSS and SFF mechanisms collinear phonons can be scattered in the [001] and [111] directions only. However, in these directions the square of the matrix element turn to zero and the FSS and SFF relaxation processes cannot be efficient mechanisms for ultrasound absorption. On the other hand, considering damping of phonon states, the FFF and SSS relaxations of collinear phonons can contribute to the ultrasound absorption if the sound wave is quasi-transverse, i.e. its longitudinal component is nonzero.

For calculating the integral  $J_{\text{FTT}}^\lambda(\theta_1, \varphi_1)$  in (20) it is necessary first to find solutions to the equation

$$\begin{aligned} F_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2) &= \cos \theta_{12} - \left( \frac{S_\lambda(\theta_1, \varphi_1)}{S_{\lambda_2}(\theta_2, \varphi_2)} \right. \\ &\left. - \Delta_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2) \right) = 0 \\ &\text{for } -1 \leq x \leq 1, \quad x = \cos \theta_2 \end{aligned} \quad (24)$$





**Figure 2.** (a) Dependences of the angle  $\theta_{12}$  between the sound wavevector and a scattered phonon ( $\theta_2, \varphi_2$ ) on the angle  $\varphi_2$  according to equation (24) in crystallographic directions like [001] ( $\theta_1 = 0, \varphi_1 = 0$ ): the FSS relaxation mechanism in Ge (curves 1, 5), Si (curves 2, 5), InSb (curves 3, 5), KCl (curves 4, 5) and CaF<sub>2</sub> (curve 5); the FFF relaxation mechanisms in Ge, Si and InSb crystals (curve 5), in KCl (curves 4', 4'' and 5) and in CaF<sub>2</sub> (curves 6 and 5); the FSS and FFF relaxation mechanisms in diamond crystals (curves 5). (b) Dependences of the angle  $\theta_{12}$  for the sound wavevectors in crystallographic directions like [101] ( $\theta_1 = \pi/4, \varphi_1 = 0$ ) on the angle  $\varphi_2$  according to equation (24): the FFF relaxation mechanisms in Ge (curve 1), Si (curve 2), InSb (curve 3), KCl (curves 4 and 4') and CaF<sub>2</sub> (curves 5 and 5').

and take the integral over  $x$  using the  $\delta$ -function (in this case, the roots of equation (24)  $x_1$  become functions of the angles  $\varphi_2, \theta_1$ , and  $\varphi_1$ ).

In the small neighborhood of  $x = x_j$  the function  $F_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)$  can be written in the form

$$F_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2) = \sum_j \frac{dF_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx} \Big|_{x=x_j} (x - x_j) + \frac{1}{2} \sum_j \frac{d^2 F_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx^2} \Big|_{x=x_j} (x - x_j)^2 + \dots \quad (25)$$

In the absence of multiple roots the derivative  $\frac{dF_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx} \Big|_{x=x_j} \neq 0$  and the expression for  $J_{\text{FTT}}(\theta_1, \varphi_1)$  becomes

$$J_{\text{FTT}}(\theta_1, \varphi_1) = \sum_{\lambda_2, j} \frac{1}{\pi} \int_0^{2\pi} d\varphi_2 \frac{I_{\text{TTT}}^{\lambda\lambda_2\lambda_2}(\theta_1, \varphi_1, x_j, \varphi_2)}{(\tilde{S}_{\lambda_2}(\theta_2, \varphi_2))^8 |F_j^{(1)}|}, \quad (26)$$

$$F_j^{(1)} = \frac{dF_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx} \Big|_{x=x_j(\varphi_2, \theta_1, \varphi_1)}.$$

If the derivative  $F_j^{(1)} = 0$ , then for the integral  $J_{\text{FTT}}(\theta_1, \varphi_1)$  we have

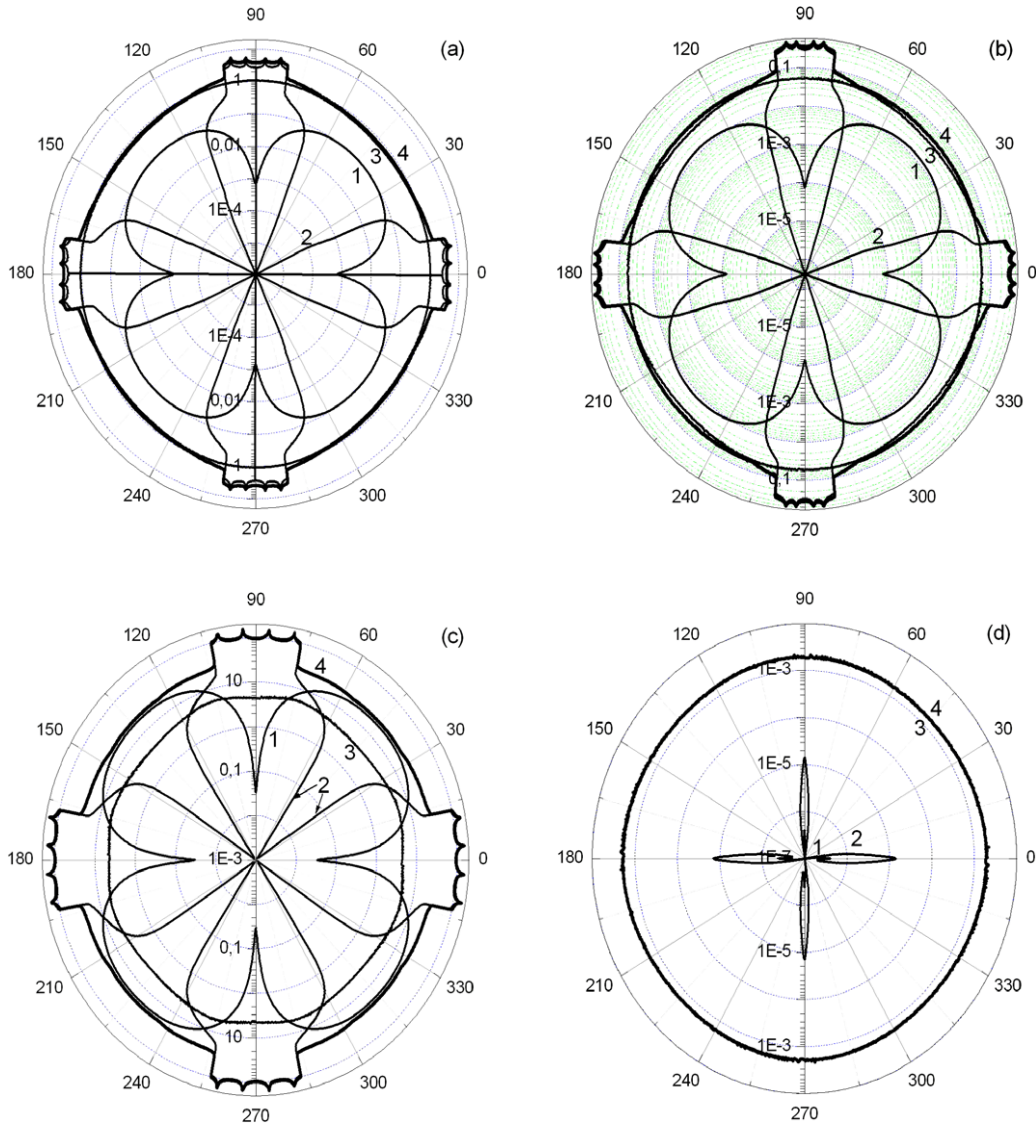
$$J_{\text{FTT}}^\lambda(\theta_1, \varphi_1) = \sum_{\lambda_2, j} \frac{2}{\pi} \int_0^{2\pi} d\varphi_2 \frac{1}{|F_j^{(2)}|} \times \frac{d}{dx} \left\{ \frac{I_{\text{TTT}}^{\lambda\lambda_2\lambda_2}(\theta_1, \varphi_1, x_j, \varphi_2)}{(\tilde{S}_{\lambda_2}(\theta_2, \varphi_2))^8} \right\} \Big|_{x=x_j}, \quad (27)$$

$$F_j^{(2)} = \frac{d^2 F_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx^2} \Big|_{x=x_j}.$$

Obviously, if the cubic anisotropy is taken into account in the energy conservation law, the procedure of deriving the roots of equation (24) can only be solved numerically. It

is necessary to examine additionally the derivative of the function  $\frac{dF_{\lambda\lambda_2}(\theta_1, \varphi_1, x, \varphi_2)}{dx} \Big|_{x=x_j} \neq 0$  and the case with multiple roots. It should be noted that  $\Delta_{\lambda\lambda_2}$  is important for the relaxation processes FSS and SFF: it ensures the interaction of noncollinear phonons in the FFF mechanism and considerably extends the interval of the angles at which long-wavelength phonons of the mode  $t_1$  can be scattered by thermal phonons of the low vibrational modes in the FSS mechanism. According to the estimates, in directions like [001] ( $\theta_1 = 0, \varphi_1 = 0$ ) and [101] ( $\theta_1 = \pi/4, \varphi_1 = 0$ ) the maximum values of  $\Delta_{t_1 t_2}$  are as large as 0.6, 0.45 and 0.17 in InSb, Ge and diamond crystals. For the FFF mechanism the maximum value of  $\Delta_{t_1 t_1}$  proves to be 1.5 times smaller: 0.4, 0.3 and 0.12 for InSb, Ge and diamond crystals, respectively. This is due to a considerably smaller anisotropy of the spectrum of the upper vibrational mode in crystals of the first group. One might expect therefore that in such elastically anisotropic crystals as Ge and InSb ( $k = 1.8$ ) the TTT mechanisms will be more significant than in diamond ( $k = 1.4$ ).

The numerical analysis of equation (24) shows that it also possesses solutions for noncollinear phonons in the case of the fast transverse mode for the FFF ( $\lambda_1 = \lambda_2 = t_1$ ) and FSS ( $\lambda_1 = t_1$  and  $\lambda_2 = t_2$ ) relaxation processes. As can be seen from figure 2(a), in directions like [001] ( $\theta_1 = 0, \varphi_1 = 0$ ) two solutions exist for the FFF relaxation processes in Ge, Si and InSb crystals: one solution corresponds to the interaction of collinear phonons ( $\theta_{12} = 0$ , figure 2(a), curves 5) and the other to the interaction of noncollinear phonons (figure 2(a), curves 1, 2 and 3). The solution corresponding to the interaction of collinear phonons (figure 3(a), curves 5) only exists for the FSS relaxation processes in these crystals. In diamond and CaF<sub>2</sub> crystals with a lower anisotropy of the harmonic energy the FFF and FSS relaxation processes in directions like [001] can only involve collinear phonons



**Figure 3.** Angular dependences of the absorptions of the fast transverse mode in Ge (a), Si (b), InSb (c) and diamond (d) crystals with wavevectors in the cube face plane ( $\varphi_1 = 0$ ): the FFF relaxation mechanism (curve 1), the FSS relaxation mechanism (curve 2), the Landau–Rumer mechanism (curve 3), and the full quasi-transverse ultrasound absorption (curve 4).

(figure 2(a), curves 5). In KCl crystals, which are more anisotropic, the energy conservation law permits the interaction of both collinear (figure 2(a), curves 5) and noncollinear (figure 2(a), curves 4, 4' and 4'') phonons. Three solutions exist for each of the two relaxation mechanisms. In directions like [101] ( $\theta_1 = \pi/4$ ,  $\varphi_1 = 0$ ) the solutions corresponding to the interaction of noncollinear phonons exist only for the FFF mechanism and are absent for the FSS relaxation mechanism in the crystals under study. It is seen from figure 2(b) that for the FFF mechanism in Ge, Si and InSb crystals equation (24) possesses two solutions corresponding to the scattering of noncollinear phonons to the angles  $0 < \theta_{12} < 25$  (curves 1, 2, 3). In KCl and CaF<sub>2</sub> crystals four solutions corresponding to the scattering of noncollinear phonons to the angles  $0 < \theta_{12} < 25$  exist for the FFF relaxation processes (see the inset in figure 2(b), curves 4, 4' and 4'').

Computation of the ultrasound absorption by finding the roots from the formulae (24)–(27) requires much machine

time. As shown in [1, 2], a simpler variant is available for calculating the relaxation rates of phonons. This variant eliminates the procedure of finding the roots of equation (24),  $F(x, \varphi_2, \theta_1, \varphi_1) = 0$ , but requires calculating a double integral  $J_{TTT}^\lambda(\theta_1, \varphi_1)$  instead of a single integral. It consists in replacement of the  $\delta$ -function in expression (20) by its representation as a limiting process from the Lorentzian or Gaussian function:

$$\delta(F(x, \varphi_2, \theta_1, \varphi_1)) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow 0} \frac{\varepsilon}{(F(x, \varphi_2, \theta_1, \varphi_1))^2 + \varepsilon^2},$$

$$\delta(F(x, \varphi_2, \theta_1, \varphi_1)) = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\sqrt{\pi\varepsilon}} \times \exp(-(F(x, \varphi_2, \theta_1, \varphi_1))^2/4\varepsilon). \quad (28)$$

The numerical analysis demonstrates that both approximations give the same result. The Gaussian approximation of the  $\delta$ -function in calculating the relaxation rates makes it possible to reduce considerably the calculation time. It should be noted

that the maximum values of the Lorentz peak are proportional to  $\varepsilon^{-1}$ , whereas those of the Gaussian peak are proportional to  $\varepsilon^{-1/2}$ ; full width at half maximum (FWHM) of the Lorentz peak is proportional to  $\varepsilon$ , whereas FWHM of the Gaussian peak is proportional to  $\varepsilon^{1/2}$ . Therefore, when calculation results depend on the degree of damping, both approximations give the same result if the parameter  $\varepsilon_L$  for calculation in the Lorentz approximation is equal to the parameter  $\varepsilon_G^2$  for calculation in the Gaussian approximation.

It should be noted that the calculated values of the ultrasound absorption for the scattering of noncollinear phonons in the variant with the damping (28) are in agreement with the exact calculations using the roots calculation procedure from formulae (24)–(27). For example, the calculated values of  $\alpha_{TLL}^\lambda(\theta_1, \varphi_1)$  for the Landau–Rumer mechanism in the variant with the damping (28) are in agreement with the exact calculations to within the error, which is not over 0.1% at  $\varepsilon = 10^{-4}$  [1]. The point is that the ratio ( $S_t/S_L$ ) for the Landau–Rumer mechanism is always less than unity. The energy conservation law is fulfilled both in the isotropic approximation and with the exact consideration of the cubic anisotropy. With this mechanism,  $\Delta_{LTT}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  is a smooth function of the angles and equation (24) possesses one or two solutions at different  $\theta_1$  and  $\varphi_1$ , while the intervals of the angles  $\varphi_2$ , over which roots are available, are sufficiently large [1]. Unlike this case, the functions having alternating sign  $\Delta_{t_1\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  for the FFF and FSS mechanisms changes abruptly with the angles  $\theta_2$  and  $\varphi_2$  over some intervals of the angles  $\theta_1$  ( $\varphi_1 = 0, \pi/4$ ). The number of roots of equation (24) is much larger than unity, while the intervals of the angles  $\varphi_2$ , over which some of the roots are available, are narrow. At  $\varphi_2 = \pi/4$  and  $\theta_2 = \theta_{111}$  the function  $\Delta_{t_1\lambda_2}(\theta_1, \varphi_1, \theta_2, \varphi_2)$  has a finite discontinuity related to the point of intersection between the spectra of quasi-transverse modes, while the function  $\frac{F(x, \varphi_2, \theta_1, \varphi_1)}{dx}$  has an infinite discontinuity at this point. Also, it turns to zero if multiple roots appear, leading to formation of sharp peaks against a smooth dependence of the ultrasound absorption. In these circumstances the numerical integration of (25) using the roots calculation procedure gives the functions  $\alpha_{FFF}(\theta_1, \varphi_1)$  and  $\alpha_{FSS}(\theta_1, \varphi_1)$  as a ‘comb’ over some intervals of the angles  $\theta_1$ . Therefore the introduction of a small, but finite, damping of the phonon states for the FFF and FSS mechanisms is a necessary procedure.

The damping of phonon states always exists. In ideal crystals the damping of phonon states is due to anharmonic scattering processes. In imperfect crystals it is due to scattering by defects and anharmonic scattering processes. According to experimental studies [24, 28–30], anharmonic processes make the dominant contribution to the ultrasound absorption at  $T > 10$  K. When  $T \sim 100$  K and  $\omega \sim 1$  GHz, the parameter  $\omega_q^\lambda \tau_2 \approx 1$ , while at higher temperatures the Landau–Rumer regime  $\omega_q^\lambda \tau_2 < 1$  is replaced by the Akhiezer regime  $\omega_q^\lambda \tau_2 > 1$  [25]. In this case, the frequency and temperature dependences of the ultrasound absorptions change qualitatively. The estimates made from measurements of the ultrasound absorptions [12–14] show that in the anharmonic scattering regime the values of the parameter  $\varepsilon \approx (\omega_q^\lambda \tau_2)^{-1}$  lie in the range  $10^{-3}$ – $10^{-1}$ . Therefore the physically reasonable

limit of the parameter  $\varepsilon \approx (\omega_q^\lambda \tau_2)^{-1}$  for the FFF and FSS mechanisms is  $10^{-6}$ , when the  $\delta$ -function is calculated as the Lorentz approximation (corresponding to  $\varepsilon_G = 10^{-12}$  in the case of the Gaussian approximation). The values of  $\alpha_{FSS}(\theta_1, \varphi_1)$  calculated in this variant for the FSS mechanism agree with those calculated by formulae (24)–(27) to within the error, which is not over 5% for the SFF mechanism; the maximum error is not over 10% for the FFF mechanism. There are two exceptions to this rule for the FFF and FSS mechanisms. Firstly, for scattering of collinear phonons in the [001] direction. In this case, the square of the matrix element (8) identically turns to zero. Therefore, if the energy conservation law is fulfilled exactly, the absorptions  $\alpha_{FFF}(\theta_1, \varphi_1)$  and  $\alpha_{FSS}(\theta_1, \varphi_1)$  turn to zero provided solutions corresponding to the interaction of noncollinear phonons are unavailable. However, the consideration of the small, but finite, damping leads to negligibly small, but finite, values of these quantities because of the small-angle scattering of phonons. Secondly, our study of the absorption of fast quasi-transverse modes by the FFF mechanism revealed a new interesting feature in the crystals of the first group. Since the spectral anisotropy of fast quasi-transverse modes proves to be smaller than that of slow quasi-transverse modes, then, for the scattering of collinear phonons ( $1-10^{-4} \leq |\cos \theta_{12}| \leq 1$ ) in some compounds under study, the function  $F_{t_1 t_1}(\theta_1, \varphi_1, x, \varphi_2)$  does not turn to zero over a narrow interval of the angles  $\theta_2$  and  $\varphi_2$ , which are close to  $\theta_1$  and  $\varphi_1$ , but takes small values of the order of  $10^{-7}$ – $10^{-8}$ . It turns out that the scattering of quasi-collinear phonons makes a finite contribution to ultrasound absorption over a wide interval of the angles  $\theta_1$  and  $\varphi_1$ . Calculations which take into account small, but finite, damping of phonon states, give much larger values of the absorption than those obtained by numerical integration (25) using the root calculation procedure. However, if the contribution of almost collinear phonons, which is found from the condition  $1-10^{-4} \leq |\cos \theta_{12}| \leq 1$ , is subtracted, results of both calculation methods agree to within 3%–5%. In what follows we shall discuss the role of the interaction between collinear, quasi-collinear and noncollinear phonons in the ultrasound absorption as applied to each of the crystals studied.

### 3. Results of the numerical analysis

From formulae (8), (9) and (18)–(28) it is possible to calculate the absorption  $A_{FTT}(\theta_1, \varphi_1)$  characterizing the dependence of the ultrasound absorption on the direction of the sound wavevector for the TTT mechanisms. We shall compare the contributions from the FFF and FSS mechanisms and the contribution from the Landau–Rumer mechanism [1], and find the full absorption of quasi-transverse ultrasound of the FT mode:

$$\begin{aligned} \alpha_{FT}(z_1, T, \theta_1, \varphi_1) &= \alpha_{FFF} + \alpha_{FSS} + \alpha_{FLL} \\ &= A_{FT}(\theta_1, \varphi_1) z_1 T^5 (\text{dB cm}^{-1}). \end{aligned} \quad (29)$$

$$\begin{aligned} A_{FT}(\theta_1, \varphi_1) &= A_{FFF}(\theta_1, \varphi_1) + A_{FSS}(\theta_1, \varphi_1) + A_{FLL}(\theta_1, \varphi_1) \\ &= A_{0FT}(J_{FFF}(\theta_1, \varphi_1) + J_{FSS}(\theta_1, \varphi_1)) + A_{0FLL} J_{FLL}(\theta_1, \varphi_1), \end{aligned}$$



$$A_{0\text{FLL}} = \frac{4.34\pi^3 k_{\text{B}}^5}{15\hbar^4 \rho^3 (S_{t_1}(\theta_1, \varphi_1))^2 \langle S_{\text{L}} \rangle^8},$$

$$A_{0\text{FT}}/A_{0\text{FLL}} = \frac{\langle S_{\text{L}} \rangle^8}{(S_{100}^t)^8} \approx \left( \frac{c_{11}}{c_{44}} \right)^4. \quad (30)$$

The quantity  $A_{\text{FT}}(\theta_1, \varphi_1)$  characterizes the anisotropy of the full absorption of the mode  $t_1$ . The corresponding dependences are calculated for the two most important cases when the phonon wavevectors lie in the planes of the cube faces or the diagonal planes. In this study we are interested in the role played by the cubic anisotropy and qualitative differences of the quasi-transverse ultrasound absorption in cubic crystals and isotropic media. The major distinction is the possibility of interaction between noncollinear phonons, leading to finite values of the absorption in the absence of the damping of phonon states. It has already been noted that in this case the absorption is zero in isotropic media. This problem is of major significance for understanding the physical processes that determine the phonon transport and the ultrasound absorption in a great variety of cubic crystals. However, in experimental conditions we always deal with damping states of thermal phonons. For this reason, in what follows we present results of the calculations, which were performed using the root computation procedure and considering a small ( $\varepsilon_{\text{L}} = 10^{-6}$ , for calculation with the Lorentz approximation of the  $\delta$ -function corresponding to  $\varepsilon_{\text{G}} = 10^{-12}$  in the case of the Gaussian approximation) damping of phonon states in accordance with the formulae (19)–(21) and (22)–(28). The calculations are made using experimental values of thermodynamic elastic moduli of the second  $c_{ik}$  and the third  $c_{ijk}$  order adopted from [4, 31] (see table 1).

Analyzing the calculation results we shall note, in the first place, the characteristic points that determine specific features of the absorption of quasi-transverse modes in cubic crystals with positive and negative anisotropies of the second-order elastic moduli. The spectrum anisotropy for the fast mode  $t_1$  is smaller than for the slow mode  $t_2$  in crystals of the first group (Ge, Si, diamond and InSb). For example, in the case of the wavevectors of phonons in planes of the cube faces the fast mode is a purely transverse isotropic mode with the polarization vector perpendicular to the cube face at hand, whereas the mode  $t_2$  is a quasi-transverse mode with the polarization vector in the cube face at hand (see [16], figure 1). Therefore relaxation mechanisms involving three transverse phonons in crystals of the first group play a greater role in the absorption of slow quasi-transverse modes than in the absorption of fast ones. In crystals of the first group the largest contribution to the absorption of fast quasi-transverse modes is due to the FSS relaxation mechanism, while the SSS relaxation mechanism dominates in the absorption of slow quasi-transverse modes [2]. The total contribution of the SSS and SFF relaxation mechanisms to the absorption of the slow mode in all crystals of the first group, except diamond, is a few times or one to two orders of magnitude larger than the contribution from the Landau–Rumer mechanism, depending on the direction (see [2], figures 3–5). On the other hand, the Landau–Rumer relaxation mechanism plays a considerable role in the absorption of fast quasi-transverse modes in Ge,

**Table 1.** Thermodynamic elasticity moduli for the cubic crystals under study, in  $10^{12}$  dyn  $\text{cm}^{-2}$ . The data are taken from [4, 31].

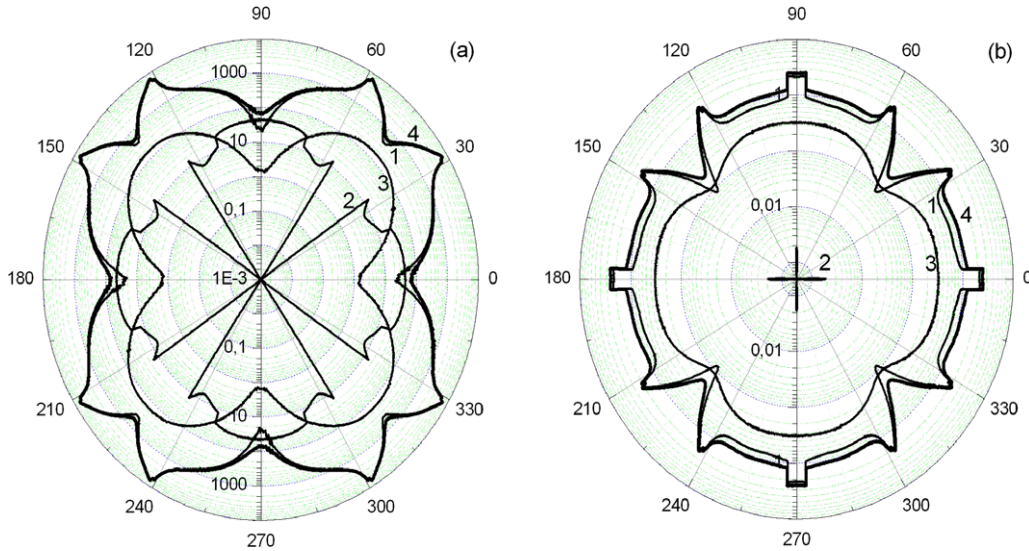
|                   | Ge    | Si    | Diamond | InSb   | KCl    | CaF <sub>2</sub> |
|-------------------|-------|-------|---------|--------|--------|------------------|
| $c_{11}$          | 1.289 | 1.657 | 10.76   | 0.672  | 0.398  | 1.644            |
| $c_{12}$          | 0.483 | 0.638 | 1.25    | 0.367  | 0.062  | 0.502            |
| $c_{44}$          | 0.671 | 0.796 | 5.758   | 0.302  | 0.0625 | 0.347            |
| $\Delta C$        | 0.54  | 0.57  | 2.01    | 0.3    | -0.211 | -0.448           |
| $k - 1$           | 0.87  | 0.67  | 0.4     | 0.81   | -0.63  | -0.35            |
| $c_{111}$         | -7.10 | -8.25 | -62.6   | -3.56  | -7.01  | -12.46           |
| $c_{112}$         | -3.89 | -4.51 | -22.6   | -2.66  | -0.571 | -4.04            |
| $c_{123}$         | -0.18 | -0.64 | 1.12    | -1.0   | 0.284  | -2.54            |
| $c_{144}$         | -0.23 | 0.12  | -6.74   | 0.16   | 0.127  | -1.24            |
| $c_{155}$         | -2.92 | -3.10 | -28.6   | -1.39  | -0.245 | -2.14            |
| $c_{456}$         | -0.53 | -0.64 | -8.23   | -0.004 | 0.118  | -0.748           |
| $\tilde{c}_{155}$ | -1.63 | -1.9  | -5.4    | -1.54  | -0.61  | 0.596            |
| $\tilde{c}_{111}$ | 28.01 | 32.4  | 138.1   | 20.96  | 1.62   | -6.71            |
| $\tilde{c}_{112}$ | -3.25 | -4.1  | -10.24  | -1.98  | -1.11  | 1.02             |

Si and diamond crystals (see tables 2 and 3). In crystals of the second group (KCl and CaF<sub>2</sub>) the spectral anisotropy for the fast mode  $t_1$  is larger than for the slow mode  $t_2$ . In the case of the wavevectors of phonons in planes of the cube faces the fast mode is a quasi-transverse mode with the polarization vector in a cube face, whereas the slow mode is a pure isotropic mode with the polarization vector perpendicular to a cube face (see [16], figure 1). Therefore the dominant contribution to the absorption of fast quasi-transverse modes is due to the FFF relaxation mechanism. The total contribution from the FFF and FSS relaxation mechanisms is much larger than the contribution from the Landau–Rumer mechanism and determines to a large extent the full absorption of fast quasi-transverse modes in KCl and CaF<sub>2</sub> crystals (see tables 2 and 3).

It should be noted that the elastic energy anisotropy determines the sound absorption efficiency of the FFF and FSS relaxation mechanisms. Firstly, it ensures the fulfilment of the energy conservation law when noncollinear phonons are scattered via the FFF and FSS relaxation mechanisms. Secondly, over the interval of the angles  $\theta_1$  and  $\varphi_1$ , at which the equation (24) possesses solutions corresponding to the scattering of noncollinear phonons, it provides for the dominance of the FFF and FSS relaxation mechanisms over the Landau–Rumer mechanism. The dominance of the FFF and FSS relaxation mechanisms over the Landau–Rumer mechanism is due in large measure to the second-order elastic moduli. As is clear from (30), the ratio of  $A_{0\text{FT}}$  and  $A_{0\text{FTL}}$  in the TTT and Landau–Rumer relaxation mechanisms is proportional to  $(c_{11}/c_{44})^4$ , which is much larger than unity. This ratio is 26, 32, 16 and 48 for the crystals of the first group (Ge, Si, diamond and InSb, respectively). Notice that the coefficient  $A_{0\text{FT}}$  is 715 and 324 times higher than  $A_{0\text{FTL}}$  in the ionic KCl and CaF<sub>2</sub> crystals, respectively. Such a large excess of  $A_{0\text{FT}}$  over  $A_{0\text{FTL}}$ , which characterizes the absorption via the Landau–Rumer mechanism, in the KCl and CaF<sub>2</sub> crystals is due to anomalously small values of the second-order elastic moduli  $c_{ik}$  determining the propagation rate of transverse phonons (see table 1).

Let us consider in more detail the absorption anisotropy for the fast quasi-transverse mode  $t_1$  when sound wavevectors are in the plane of a cube face ( $\varphi_1 = 0$ ) (see figures 3 and 4).





**Figure 4.** Angular dependences of the absorption of the fast quasi-transverse mode in the KCl (a) and CaF<sub>2</sub> (b) crystals with the wavevector in the cube face plane ( $\varphi_1 = 0$ ): the FFF relaxation mechanism (curve 1), the FSS relaxation mechanism (curve 2), the Landau–Rumer mechanism (curve 3), and full quasi-transverse ultrasound absorption (curve 4).

**Table 2.** Parameters determining quasi-transverse ultrasound absorption for FFF and FSS relaxation mechanisms in the crystals under study.

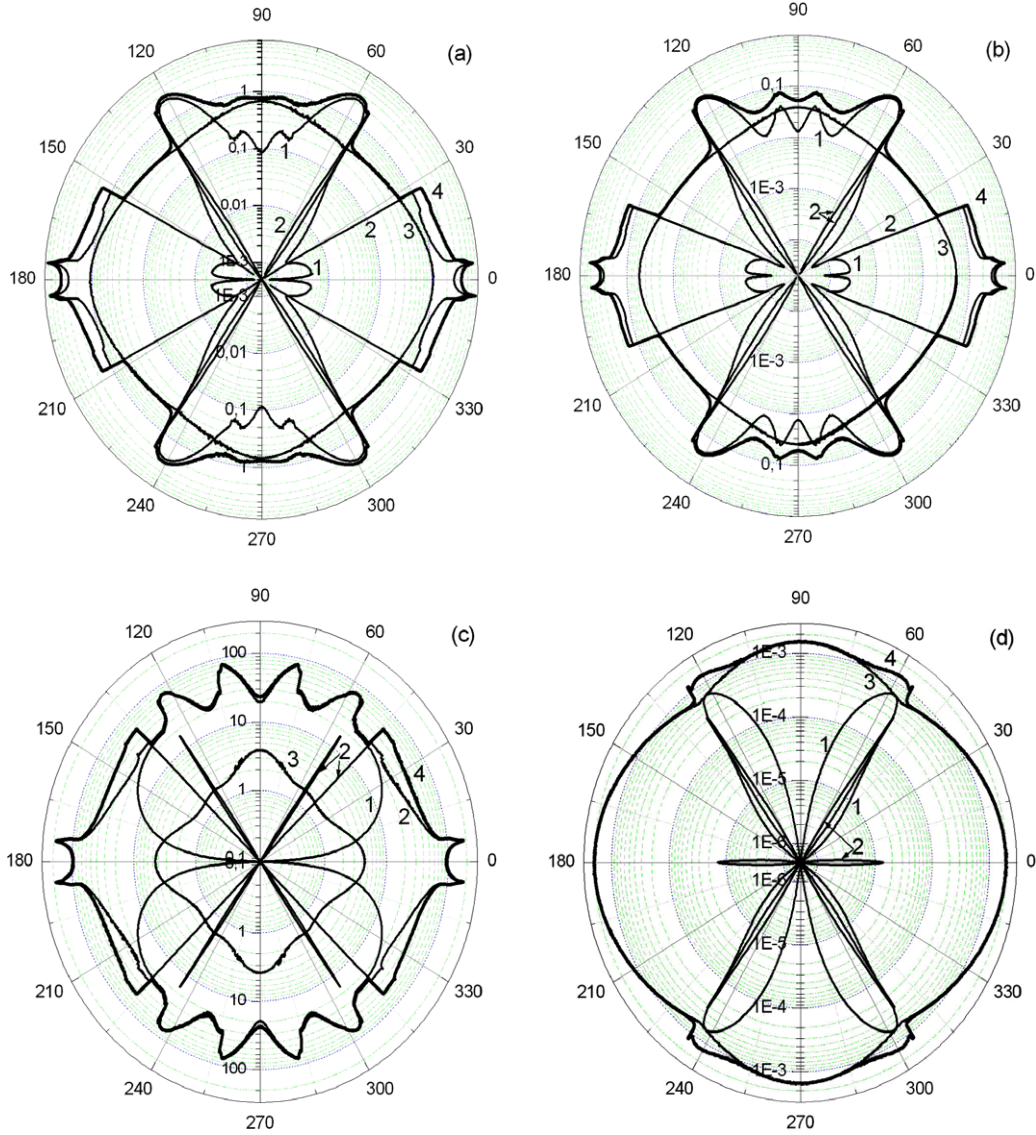
|                  | $A_{0FT} \times 10^5$<br>(db K <sup>-5</sup> cm <sup>3</sup> dyn <sup>-2</sup> ) | $A_{FFF}(\theta_1, \varphi_1) \times 10^5$ (db cm <sup>-1</sup> K <sup>-5</sup> ) |       |                      | $A_{FSS}(\theta_1, \varphi_1) \times 10^5$ (db cm <sup>-1</sup> K <sup>-5</sup> ) |                    |                       |
|------------------|--|---|-------|----------------------|---|--------------------|-----------------------|
|                  |  | [100]   | [101] | [111]                | [001]   | [101]              | [111]                 |
| Ge               | 7.59   | $7.1 \times 10^{-4}$  | 0.11  | 1.23                 | 2.74  | 0                  | 0.64                  |
| Si               | 0.62   | $7.5 \times 10^{-5}$  | 0.013 | 0.12                 | 0.3   | 0                  | $6.11 \times 10^{-2}$ |
| Diamond          | $7.1 \times 10^{-5}$   | $4 \times 10^{-7}$  | 0     | $3.5 \times 10^{-4}$ | $1.4 \times 10^{-5}$  | 0                  | $2.2 \times 10^{-4}$  |
| InSb             | 480.6  | 0.03  | 19.4  | 59.32                | 92.12   | 0                  | 13.74                 |
| KCl              | $1.5 \times 10^5$  | 24.28   | 415.2 | 522.78               | 43.197  | $1 \times 10^{-3}$ | 581.61                |
| CaF <sub>2</sub> | 73.2   | 2.14  | 0.083 | 0.14                 | 0.0018  | 0                  | $3.58 \times 10^{-2}$ |

It is seen from these figures that the angular dependences of the absorption of the fast quasi-transverse mode and the contributions from the FFF and FSS relaxation mechanisms are qualitatively different in cubic crystals of the first ( $\Delta C > 0$ , Ge, Si, diamond and InSb) and second (KCl and CaF<sub>2</sub>) types. In the case under consideration ( $\varphi_1 = 0$ ) the angular dependences of the absorptions  $\alpha_{FFF}(\theta_1, 0)$ ,  $\alpha_{FSS}(\theta_1, 0)$  and  $\alpha_{FT}(\theta_1, 0)$  are qualitatively similar in crystals of the first type (Ge, Si and InSb) (see figures 3). However, the ratios of the contributions from the FFF, FSS and Landau–Rumer relaxation mechanisms are considerably different (see tables 2 and 3). In directions like [001] and directions close to them (at  $\pi/2n - 0.17 < \theta_1 < \pi/2n + 0.17$ ) the FSS relaxation mechanism dominates in Ge, Si and InSb crystals, while the absorptions  $\alpha_{FT}(\theta_1, 0)$  and  $\alpha_{FSS}(\theta_1, 0)$  have similar values (see figures 3(a)–(c), curves 1 and 4). The contribution of the FFF relaxation mechanism is small in Ge and Si crystals, while the Landau–Rumer mechanism dominates over a wide interval of the angles ( $0.17 < \theta_1 < 1.4$  in Ge and  $0.16 < \theta_1 < 1.41$  in Si) and determines to a large extent the ultrasound absorption (see figures 3(a) and (b), curves 1, 3 and 4). On the other hand, in InSb the contribution of the Landau–Rumer mechanism to the full absorption is small at all angles  $\theta_1$  and the full absorption is determined mainly by the total contribution of

**Table 3.** Parameters determining quasi-transverse ultrasound absorption for Landau–Rumer relaxation mechanism in the crystals under study.

|                  | $A_{00FLL} \times 10^5$<br>(db K <sup>-5</sup> cm <sup>3</sup> dyn <sup>-2</sup> ) | $A_{FLL}(\theta_1, \varphi_1) \times 10^5$ (db cm <sup>-1</sup> K <sup>-5</sup> ) |         |                       |
|------------------|--|---|---------|-----------------------|
|                  |  | [100]   | [101]   | [111]                 |
| Ge               | 0.29   | 1.06  | 0.68    | 0.32                  |
| Si               | 0.019  | 0.05  | 0.04    | 0.023                 |
| Diamond          | $4.4 \times 10^{-6}$   | 0.00194   | 0.00153 | $6.15 \times 10^{-4}$ |
| InSb             | 9.91   | 4.41  | 3.93    | 1.76                  |
| KCl              | 210  | 1.53  | 156.71  | 136.7                 |
| CaF <sub>2</sub> | 0.23   | 0.33  | 0.114   | 0.2                   |

the FFF and FSS relaxation mechanisms. The dominant contribution to the full ultrasound absorption is made by the FSS relaxation mechanism in directions close to [001] and the FFF mechanism in directions close to [101]. In [001] directions the absorptions  $\alpha_{FT}(\theta_1, 0)$  and  $\alpha_{FSS}(\theta_1, 0)$  have a local minimum and reach the maximum values at the angles  $\theta_1 = \pi/2n \pm 0.13$ ,  $\theta_1 = \pi/2n \pm 0.09$  and  $\theta_1 = \pi/2n \pm 0.06$  in Ge, Si and InSb crystals, respectively. In the case of the FSS relaxation mechanism the energy conservation law allows the interaction of both collinear and noncollinear



**Figure 5.** Angular dependences of the absorption of the fast quasi-transverse mode in the Ge (a), Si (b), InSb (c) and diamond (d) crystals with the wavevector in the diagonal plane ( $\varphi_1 = \pi/4$ ): the FFF relaxation mechanism (curve 1), the FSS relaxation mechanism (curve 2), the Landau–Rumer mechanism (curve 3), and full quasi-transverse ultrasound absorption (curve 4).

phonons in directions like [001] (see figure 2(a)). However, the contribution to the absorption  $\alpha_{\text{FSS}}(0, 0)$  is determined by the interaction of noncollinear phonons. It is 3, 6 and 21 times larger than the contribution of the Landau–Rumer mechanism in Ge, Si and InSb crystals, respectively. At the angles  $\pi/2n + 0.4 < \theta_1 < \pi/2(n + 1) - 0.4$  in Ge,  $\pi/2n + 0.3 < \theta_1 < \pi/2(n + 1) - 0.3$  in Si and  $\pi/2n + 0.6 < \theta_1 < \pi/2(n + 1) - 0.6$  in InSb equation (24) does not possess solutions corresponding to the interaction of noncollinear phonons in the FSS mechanism and the absorption  $\alpha_{\text{FSS}}(\theta_1, 0)$  turns to zero. The solution to equation (24) suggests that in directions like [001] the FFF relaxation mechanism can involve collinear phonons only ( $\theta_{12} = 0$ ). In this case the square of the matrix element (8) identically turns to zero. Therefore, if the energy conservation law is fulfilled exactly, the absorption of the fast quasi-transverse mode  $\alpha_{\text{FFF}}(0, 0)$  becomes zero. However, taking into account the small damping ( $\varepsilon_L = 10^{-6}$ )

leads to a negligibly small, but finite value of the absorption  $\alpha_{\text{FFF}}(0, 0)$ . In the case of the FFF relaxation mechanism the absorptions  $\alpha_{\text{FFF}}(\theta_1, 0)$  are a maximum in directions like [101]. In this direction the energy conservation law allows the interaction of both collinear and noncollinear phonons. The region of existence of the roots corresponding to the interaction of noncollinear phonons is much larger in InSb than in Ge and Si (see figure 2(b), curves 1, 2 and 3). For this reason the FFF mechanism is significant for InSb crystals, but makes a small contribution in Ge and Si crystals. The absorption  $\alpha_{\text{FFF}}(\pi/4, 0)$  is 6 and 3 times smaller than the contribution from the Landau–Rumer mechanism in Ge and Si, respectively, whereas in InSb crystals  $\alpha_{\text{FFF}}(\pi/4, 0)$  is 5 times larger than  $\alpha_{\text{FLL}}(\pi/4, 0)$ . Thus, with the domination of anharmonic processes of the scattering, taking into account one of the FFF, FSS and Landau–Rumer relaxation processes is insufficient for the quantitative description of the absorption anisotropy in

Ge, Si and InSb crystals. For example, the contributions of the FFF, FSS and Landau–Rumer relaxation mechanisms to the full absorption in directions like [001] in crystals of the first type are 0.2%, 72% and 27.8% in Ge, 0.01%, 85.79% and 14.2% in Si, and 0.03%, 95.4% and 4.57% in InSb, respectively. In directions like [101] they are 14%, 0% and 86% in Ge, 20%, 0% and 80% in Si and 83.3%, 0% and 16.7% in InSb, respectively. The ultrasound absorption anisotropy is characterized by the ratio of  $\alpha_{FT}(0, 0)$  and  $\alpha_{FT}(\pi/4, 0)$ . This ratio equals 5, 7 and 4 in Ge, Si and InSb, respectively, and differs considerably from the corresponding ratios in the case of the Landau–Rumer mechanism (see tables 3 and 4). Thus, the FFF and FSS relaxation mechanisms are important for the crystals of the first type under study: they determine to a large degree the anisotropy of the full absorption of the fast transverse mode.

The cubic anisotropy parameter  $k - 1$  is much smaller in diamond than it is in Ge and InSb crystals and, hence, the TTT relaxation mechanisms are less significant for diamond (see table 1). The full absorption  $\alpha_{FT}(\theta_1, 0)$  is determined to a large extent by the Landau–Rumer mechanism (see figure 3(d)). The solution to equation (24) for sound wavevectors lying in the cube face plane ( $\varphi_1 = 0$ ) suggests that the FFF and FSS relaxation mechanisms can only involve collinear phonons ( $\theta_{12} = 0$ ). Therefore, if the energy conservation law is fulfilled exactly, the absorptions  $\alpha_{FFF}(0, 0)$  and  $\alpha_{FSS}(0, 0)$  turn to zero. However, the consideration of the small damping leads to negligibly small, but finite, values of these quantities because of the small-angle scattering. For example, in the [001] directions at  $\varepsilon = 10^{-6}$  the contribution of the Landau–Rumer mechanism to the full absorption is two orders of magnitude larger than the total contribution from the SSS and SFF relaxation mechanisms. So the full absorption  $\alpha_{FT}(\theta_1, 0)$  is determined by the Landau–Rumer mechanism.

In KCl and CaF<sub>2</sub> cubic crystals with a negative anisotropy of the second-order elastic moduli the ratio of the contributions from the FFF and FSS relaxation mechanisms to the ultrasound absorption is considerably different from the corresponding ratio in crystals of the first group (see figures 3 and 4). In the latter crystals the absorptions  $\alpha_{FFF}(\theta_1, 0)$  and  $\alpha_{FSS}(\theta_1, 0)$  are due to the interaction of noncollinear phonons and the FFF mechanism makes the dominant contribution to the full absorption  $\alpha_{FT}(\theta_1, 0)$  over a wide interval of the angles  $\theta_1$ . As indicated above, this is due to the fact that in crystals of the second group the anisotropy of the fast mode is much larger than the anisotropy of the slow mode (see [16], figure 1). Therefore the function  $\Delta_{t_1 t_1}(\theta_1, \varphi_1, \theta_2, \varphi_2)$ , which is proportional to angular derivatives of the sound phase velocity, takes larger values than for the  $t_2$  mode. In this case the interval of angles at which noncollinear phonons can interact is considerably extended. KCl crystals are the most anisotropic among crystals of the second group. The FFF relaxation mechanism determines to a large degree the full absorption of the fast mode in KCl: the curves 1 and 4 in figure 4(a) are similar over a wide interval of angles  $\theta_1$ . It is only in the vicinity of directions like [001] that the FSS relaxation mechanism makes an appreciable contribution to the full absorption. In this case the absorption  $\alpha_{FSS}(0, 0)$  is almost

**Table 4.** Parameters determining full quasi-transverse ultrasound absorption in the crystals under study.

|                  | $A_{FT}(\theta_1, \varphi_1) \times 10^5$ (db cm <sup>-1</sup> K <sup>-5</sup> ) |          |                       | $A_{0FT}/A_{0FLL}$ |
|------------------|--|----------|-----------------------|--------------------|
|                  | [100]  | [101]    | [111]                 |                    |
| Ge               | 3.81   | 0.78     | 2.19                  | 26                 |
| Si               | 0.36   | 0.05     | 0.204                 | 32                 |
| Diamond          | 0.001 95   | 0.001 53 | $11.9 \times 10^{-4}$ | 16                 |
| InSb             | 96.56  | 23.3     | 74.82                 | 48                 |
| KCl              | 69.01  | 571.9    | 1241.1                | 715                |
| CaF <sub>2</sub> | 2.47   | 0.197    | 0.37                  | 324                |

twice as large as  $\alpha_{FFF}(0, 0)$  and more than 30 times larger than the contribution from the Landau–Rumer mechanism  $\alpha_{FLL}(0, 0)$  (see figure 4(a), curves 1, 2 and 3). In KCl the full absorption  $\alpha_{FT}(\theta_1, 0)$  is a maximum at the angles  $\theta_1 \cong n\pi/2 \pm 0.55$  and  $\theta_1 \cong n\pi/2 \pm 1.02$  ( $n = 0, 1, 2$ , etc), and a minimum in directions like [001]. A local minimum is realized in [101] directions ( $\theta_1 = \pi/4$ ) (see figure 4(a), curves 4). The contributions from the FFF, FSS and Landau–Rumer relaxation mechanisms to the full ultrasound absorption equal 35.2%, 62.6% and 2.2% in directions like [001] and 72.6%, 0% and 27.4% in directions like [101] (see tables 2–4). Thus, the Landau–Rumer mechanism makes a relatively small contribution to the absorption of the fast quasi-transverse mode.

In contrast to KCl, in CaF<sub>2</sub> crystals the Landau–Rumer mechanism is more significant for the ultrasound absorption. The point is that the elastic energy anisotropy in CaF<sub>2</sub> is much smaller than in KCl: the cubic anisotropy parameter  $|k - 1|$  is twice as low as in KCl (see table 1). Although the contribution of the Landau–Rumer mechanism to the absorption in [001] directions is one order of magnitude smaller than the contribution of the FFF mechanism, in directions like [101] the absorption  $\alpha_{FLL}(\pi/4, 0)$  is 1.4 times higher than  $\alpha_{FFF}(\pi/4, 0)$ . The contribution of the FSS relaxation mechanism to the full absorption is negligibly small in CaF<sub>2</sub> crystals. In directions like [001], collinear phonons ( $\theta_{12} = 0$ ) can only participate in this mechanism. Therefore, if the energy conservation law is fulfilled exactly, the absorption  $\alpha_{FSS}(0, 0)$  becomes zero in CaF<sub>2</sub> crystals. However, if the small damping is taken into account, it becomes nonzero due to the small-angle scattering. At  $\varepsilon_L = 10^{-6}$  the absorption  $\alpha_{FSS}(0, 0)$  is two orders of magnitude lower than the contribution of the Landau–Rumer mechanism  $\alpha_{FLL}(0, 0)$  (see figure 4(b), curves 2 and 3). The contributions of the FFF, FSS and Landau–Rumer relaxation mechanisms to the full absorption equal 86.6%, 0.1% and 13.3%, respectively, in directions like [001] and 42.1%, 0% and 57.9%, respectively, in directions like [101]. In CaF<sub>2</sub> crystals the full absorption  $\alpha_{FT}(\theta_1, 0)$  is a maximum at the angles  $\theta_1 \cong n\pi/2 \pm 0.57$  and  $\theta_1 \cong n\pi/2 \pm 1.01$  ( $n = 0, 1, 2$ , etc) and a minimum in directions like [101] (see figure 4(b), curve 4). A local maximum is realized in [001] directions. It should be noted that the anisotropy of the full absorption  $\alpha_{FT}(\theta_1, 0)$  is considerably different from the ultrasound absorption anisotropy in the Landau–Rumer mechanism. For example, the ratio of  $\alpha_{FT}(\pi/4, 0)$  and  $\alpha_{FT}(0, 0)$  is 8 and 0.1 for KCl and CaF<sub>2</sub>,



respectively. In the case of the Landau–Rumer mechanism the ratio of  $\alpha_{\text{FLL}}(\pi/4, 0)$  and  $\alpha_{\text{FLL}}(0, 0)$  is 102 and 0.35 for KCl and CaF<sub>2</sub>, respectively (see table 3).

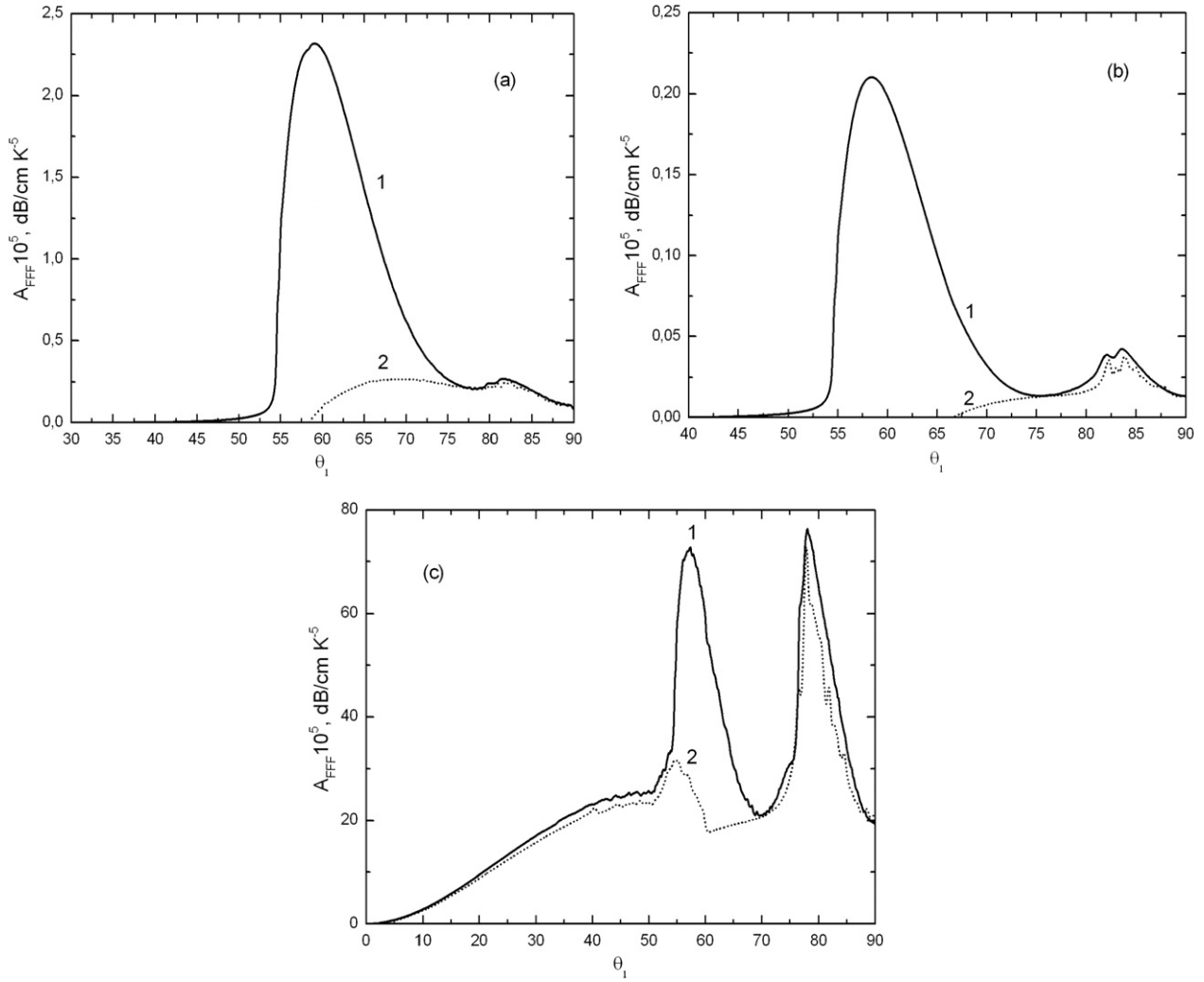
The angular dependences of the absorptions  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$ ,  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  in the crystals of the first and second types are more complicated for sound wavevectors lying in the diagonal plane ( $\varphi_1 = \pi/4$ ). This is due to both the presence of the intersection point of the spectra of quasi-transverse modes and the behavior of the polarization vectors of the fast mode [16]. As mentioned above, in crystals of the first type the fast mode  $t_1$  is a pure mode with the polarization vector perpendicular to the diagonal plane at the angles  $0 < \theta_1 < \theta_{111}$  and  $\pi - \theta_{111} < \theta_1 < \pi$ , while it is a quasi-transverse mode with the polarization vector lying in the diagonal plane at the angles  $\theta_{111} < \theta_1 < \pi - \theta_{111}$ . In the crystals of the second type these intervals of the angles change places (for details see [16]). In this connection, the angular dependences of the absorptions  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$ ,  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  in directions like [111] exhibit singularities: sharp local maxima are observed in the all crystals of the first type (Ge, Si, diamond and InSb) and the second type (KCl and CaF<sub>2</sub>).

As with ( $\varphi_1 = 0$ ), in this case ( $\varphi_1 = \pi/4$ ) the angular dependences of the absorptions  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$ ,  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  are qualitatively similar. However, the ratio of the contributions from the FFF, FSS and Landau–Rumer relaxation mechanisms to the full absorption in Ge and Si crystals is considerably different from the ratio in InSb (see figures 5(a)–(c)). The FSS mechanism determines to a large degree the full absorption in directions close to [001] at the angles  $-0.5 < \theta_1 < 0.5$  and  $-0.42 < \theta_1 < 0.42$  in Ge and Si, respectively. In Ge and Si crystals the FFF mechanism makes the dominant contribution to the full absorption at the angles  $0.5 < \theta_1 < 0.96$  and  $0.96 < \theta_1 < 1.25$  in Ge and  $0.42 < \theta_1 < 0.96$  and  $1.24 < \theta_1 < \pi/2$  in Si. On the other hand, in InSb the total contribution of the FFF and FSS relaxation mechanisms is much larger than the contribution of the Landau–Rumer mechanism to the full absorption over the whole interval of the angles  $\theta_1$ . In this crystal the full absorption is determined by the contribution from the FSS relaxation mechanism at the angles  $0 < \theta_1 < \pi/4$  and the FFF mechanism at the angles  $\pi/4 < \theta_1 < \pi/2$ . Local minima of the full absorption  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  are realized in [001] directions in Ge, Si and InSb. Maximum values of  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  are due to the FSS relaxation mechanism and are achieved at angles close to directions like [001]:  $\theta_{1\text{max}} = 0.07, 0.05$  and  $0.09$  in Ge, Si and InSb, respectively. These values are 1.7, 1.6 and 1.9 times larger than  $\alpha_{\text{FT}}(0, \pi/4)$  in Ge, Si and InSb, respectively. The absorption  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  is a minimum at  $\theta_1 \cong 0.9$  in Ge and Si crystals and  $\theta_1 = \pi/2$  in InSb. The contributions from the FFF, FSS and Landau–Rumer relaxation mechanisms to the full absorption in directions like [111] in crystals of the first type are 56.2%, 29.2% and 14.6% in Ge, 58.8%, 30% and 11.2% in Si and 79.3%, 18.4% and 2.3% in InSb. Thus, the quantitative description of the absorption anisotropy of fast quasi-transverse modes in Ge, Si and InSb crystals should take into account both the FFF and FSS relaxation mechanisms and the Landau–Rumer mechanism.

Analyzing the absorption of fast quasi-transverse modes in the FFF mechanism, we detected an interesting new feature of Ge, Si, diamond and InSb crystals. The calculation of the absorption  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  for these crystals taking into account the small damping of the phonon states ( $\varepsilon_L = 10^{-6}$ ) in accordance with the formulae (18)–(21) and (28) gives much larger absorption values than those obtained by the numerical integration using the root calculation procedure (see figures 5(d) and 6, curves 1 and 2). This feature appears because the spectral anisotropy of the fast quasi-transverse modes is insufficient for equation (24) to possess a solution. Therefore, when quasi-collinear phonons are scattered in Ge, Si, diamond and InSb crystals, the function  $F_{t_1 t_1}(\theta_1, \varphi_1, x, \varphi_2)$  does not turn to zero over the interval of angles  $1 - 10^{-4} \leq |\cos \theta_{12}| \leq 1$ . However, over a narrow interval of the angles  $\theta_2$  and  $\varphi_2$  approaching  $\theta_1$  and  $\varphi_1$  it takes small values (about  $10^{-7}$ – $10^{-8}$ ). In this case the scattering of quasi-collinear phonons makes a finite contribution to the ultrasound absorption in directions close to [111] in diamond crystals and over a sufficiently wide interval of the angles  $\theta_1$  in Ge, Si and InSb. Let us dwell in more detail on diamond crystals, where the elastic energy anisotropy is much smaller than it is in Ge, Si and InSb crystals. It follows from the analysis of equation (24) that the FFF and FSS relaxation mechanisms can involve collinear phonons ( $\theta_{12} = 0$ ) only. Therefore, if the energy conservation law is fulfilled exactly, the absorptions  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  become zero at all angles  $\theta_1$ . Taking into account the small damping leads to negligibly small, but finite, values of these absorptions on account of the small-angle scattering. This case is realized in [001] and similar directions. The total contribution of the FFF and FSS relaxation mechanisms to the full absorption is small and can be neglected: it is two orders of magnitude smaller than the contribution of the Landau–Rumer mechanism at  $\varepsilon_L = 10^{-6}$ . However, in diamond the absorptions  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  in directions close to [111] have specific features related to the scattering of quasi-collinear phonons. As a result, in directions close to [111] the angular dependences of  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  and  $\alpha_{\text{FSS}}(\theta_1, \pi/4)$  exhibit peaks comparable in value with the contribution from the Landau–Rumer mechanism (see figure 5(d), curves 1 and 2). Thus, the Landau–Rumer mechanism makes the dominant contribution to the ultrasound absorption in diamond crystals, while it is only in directions close to [111] that the features related to the scattering of quasi-collinear phonons appear in the dependence of the full absorption.

The scattering of quasi-collinear phonons makes a more substantial contribution to the full absorption in Ge, Si and InSb crystals than in diamond (see figures 6(a)–(c), curves 1 and 2). It is seen from figure 6 that the scattering leads to finite values of the absorption  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  over the interval of the angles  $\theta_1$  at which the roots corresponding to the interaction of noncollinear phonons are unavailable. In Ge, Si and InSb crystals the FFF mechanism involving quasi-collinear phonons makes the dominant contribution to the full absorption  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  at the angles  $0.96 < \theta_1 < 1.25$ . It determines the appearance of local maxima in  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  curves at  $\theta_1 \cong \pi/3$  (see figures 5(a)–(c), curves 1 and 4). The



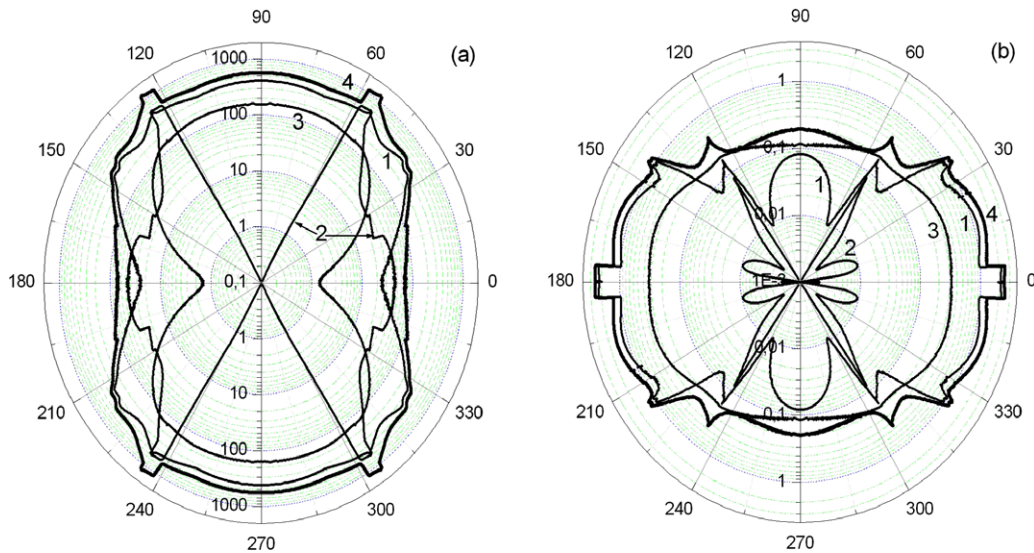


**Figure 6.** Angular dependences of the absorption of the fast quasi-transverse mode for the FFF relaxation mechanism in Ge (a), Si (b) and InSb (c) crystals with the wavevector in the diagonal plane ( $\varphi_1 = \pi/4$ ): curves 1, the calculations of the absorption taking into account the small damping of the phonon states ( $\varepsilon_L = 10^{-6}$ ); curves 2, the calculations obtained by the numerical integration using the root calculation procedure in accordance with the formulae (18)–(21) and (24)–(28).

analysis demonstrates that results of the numerical integration with the root calculation procedure agree to within 3% with the calculated absorption  $\alpha_{\text{FFF}}(\theta_1, \pi/4)$  in Ge, Si and InSb crystals taking into account the damping of phonon states determined from the condition  $1 - 10^{-4} \leq |\cos \theta_{12}| \leq 1$ . Thus, taking into account the damping of phonon states in relaxation mechanisms involving three transverse phonons can lead to appearance of additional features of the ultrasound absorption.

In KCl and  $\text{CaF}_2$  crystals the absorption anisotropy for the wavevectors in the diagonal plane ( $\varphi_1 = \pi/4$ ) is much larger than for the wavevectors in the plane of a cube face (see figures 7, curves 4). The ratios of the contributions from the FFF, FSS and Landau–Rumer relaxation mechanisms in these crystals are considerably different. In KCl crystals the total contribution of the FFF and FSS relaxation mechanisms to the full absorption is much larger than the contribution of the Landau–Rumer mechanism. The FFF mechanism dominates over a wide interval of angles  $\theta_1$ . The FSS relaxation mechanism dominates only in directions close to [001]: the

absorption  $\alpha_{\text{FSS}}(0, \pi/4)$  is 1.8 times higher than  $\alpha_{\text{FFF}}(0, \pi/4)$  (see figures 7, curves 1 and 2). Contrastingly, in  $\text{CaF}_2$  crystals the contribution of the FSS relaxation is small compared to the contributions from the FFF and Landau–Rumer relaxation mechanisms. The FFF mechanism dominates at angles  $0.7 < \theta_1 < -0.7$  and the Landau–Rumer mechanism at angles  $0.7 < \theta_1 < 2.27$ . In KCl crystals the absolute maximum of the full absorption  $\alpha_{\text{FT}}(\theta_1, \pi/4)$  is due to the FFF and FSS relaxation mechanisms and is realized at  $\theta_1 \cong 0.95$ . In  $\text{CaF}_2$  crystals they are due to the FFF relaxation mechanisms and are realized in the [001] directions. The full absorption reaches its minimum values in directions like [001] in KCl and at  $\theta_1 \cong 1.16$  in  $\text{CaF}_2$ . The minimum values are determined by the FFF and FSS relaxation mechanisms in KCl crystals and the Landau–Rumer mechanism in  $\text{CaF}_2$  crystals. The maximum values are 18.4 and 15 times larger than the minimum ultrasound absorptions in KCl and  $\text{CaF}_2$ , respectively. The contributions of the FFF, FSS and Landau–Rumer relaxation mechanisms to the full absorption in directions like [111] are 42.1%, 46.9%



**Figure 7.** Angular dependences of the absorption of the fast quasi-transverse mode in the KCl (a) and CaF<sub>2</sub> (b) crystals with the wavevector in the diagonal plane ( $\varphi_1 = \pi/4$ ): the FFF relaxation mechanism (curve 1), the FSS relaxation mechanism (curve 2), the Landau–Rumer mechanism (curve 3), and full quasi-transverse ultrasound absorption (curve 4).

and 11% in KCl crystals and 37.8%, 9.6% and 52.6% in CaF<sub>2</sub> crystals.

Thus, the absorption coefficients change over sufficiently wide limits in the crystals under study. The absorption is a maximum in the KCl crystal among those studied: the coefficient  $A_{FT}(\theta_1, \pi/4)$  is as high as 1272 (dB cm<sup>-1</sup> K<sup>-5</sup>) at the angle  $\theta_1 \cong 0.95$ . The absorptions are a minimum in the diamond crystals: coefficient  $A_{FT}(\theta_1, 0) = 0.00067$  (dB cm<sup>-1</sup> K<sup>-5</sup>) at the angle  $\theta_1 \cong 0.92$ .

#### 4. Conclusion

The absorption of quasi-transverse ultrasound during anharmonic scattering processes in cubic crystals with a positive (Ge, Si, diamond and InSb) or a negative (KCl and CaF<sub>2</sub>) anisotropy of the second-order elastic moduli was studied. The absorption of the fast quasi-transverse modes via the FFF, FSS and Landau–Rumer relaxation mechanisms was discussed. The angular dependences of the absorptions of the fast quasi-transverse modes for the FFF and FSS relaxation mechanisms were analyzed in the context of the anisotropic continuum model and the full absorptions were determined. Two most important cases were considered: when the wavevectors of phonons are in the planes of the cube faces or in the diagonal planes. The main results of the study can be formulated as follows:

(1) The absorption of quasi-transverse ultrasound in cubic crystals is calculated for the FFF and FSS relaxation mechanisms in a long-wavelength approximation. It is shown that in crystals with a considerable anisotropy of the elastic energy (InSb and KCl) the total contribution from the FFF and FSS relaxation mechanisms to the ultrasound absorption is one to two orders of magnitude larger than the contribution from the Landau–Rumer mechanism depending on the direction. The dominance

of the FFF and FSS relaxation mechanisms over the Landau–Rumer mechanism is explained, to a large extent, by the second-order elastic moduli. The role of the Landau–Rumer mechanism in the ultrasound absorption is considerable in cubic crystals with a smaller anisotropy of the elastic energy.

- (2) It is shown that the FFF and FSS relaxation mechanisms are due to the cubic anisotropy of the crystals leading to the interaction of noncollinear phonons. With the anharmonic scattering processes playing the dominant role, the consideration of one of the relaxation mechanisms—the Landau–Rumer mechanism or the FFF or FSS mechanism—is insufficient for the quantitative description of the anisotropy of full ultrasound absorption in cubic crystals.
- (3) The full absorption of fast quasi-transverse modes is determined. It is shown that other variants of the FT mode relaxation, except the ones considered above, leading to the dependence of the Landau–Rumer type are unavailable.
- (4) The analysis of three-phonon scattering processes via the FFF and FSS relaxation mechanisms in cubic crystals demonstrated that the square of the matrix element turns to zero during the scattering of collinear phonons when the fast FT mode is a purely transverse mode and is nonzero when the fast FT mode is quasi-transverse. The behavior of the square of the matrix element during the scattering of collinear phonons is qualitatively different in crystals of the first and second groups.
- (5) It is found that the presence of intersection points of the spectra of quasi-transverse modes and a sharp change of the polarization vectors near the [111] directions give rise to sharp local maxima in the absorptions of fast quasi-transverse modes in the vicinity of these directions.

(6) New features of ultrasound absorption in the FFF mechanism, which arise from the scattering of quasi-collinear phonons, have been detected in Ge, Si, diamond and InSb crystals. In diamond crystals the scattering of quasi-collinear phonons leads to appearance of peaks in the absorption curve in directions close to [111]. The values of these peaks are comparable with the contribution from the Landau–Rumer mechanism. In Ge, Si and InSb crystals this mechanism makes the dominant contribution to the full absorption  $\alpha_{FT}(\theta_1, \pi/4)$  over a sufficiently wide interval of the angles  $\theta_1$  and determines the appearance of local maxima in the  $\alpha_{FT}(\theta_1, \pi/4)$  curves.

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